General Relativistic Simulations of Cosmic Large Scale Structure

based on arXiv:1308.6524

and work in progress with C. Clarkson, E. DiDio, R. Durrer, and M. Kunz

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W3: Origin of Cosmic Structures

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Outline

Motivation: A Case for GR

- Newton vs. Einstein
- The Issue of Backreaction
- Relativistic Sources of Stress-Energy

The Framework

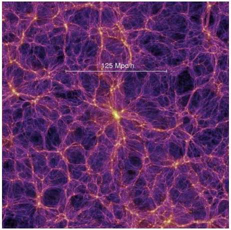
- Choice of Variables
- Weak Field Approximation
- System of Equations
- Algorithmic Solutions

Numerical Results

- A Plane-Symmetric Setup

A Case for GR

Newton vs. Einstein



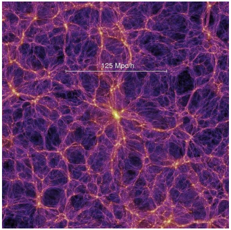
Millennium simulation,
Springel *et al.* 2005

In order to study the regime of nonlinear structure formation, large N-body simulations are the method of choice.

N-body simulations use
Newton's law of gravity

A Case for GR

Newton vs. Einstein



Millennium simulation,
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N-body simulations use
Newton's law of gravity

Works well for *nonrelativistic matter* (CDM), because of
see, e.g., Green & Wald 2012

- Exact correspondence between Newtonian gravity and GR on the background solution (FRW)
- Exact correspondence also on the level of linear (scalar) perturbations
- Nonlinear scale $\ll$ Hubble scale

Newton vs. Einstein (cont.)

The Newtonian picture has several drawbacks, though

- Strong assumtion about material content of the Universe
- Misses some degrees of freedom (gravity waves!)
- Gauge issues are not apparent
- Trivial propagation of light beams (relativistic effects have to be put back "by hand")

A Case for GR

Newton vs. Einstein (cont.)

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- Strong assumtion about material content of the Universe
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- Gauge issues are not apparent
- Trivial propagation of light beams (relativistic effects have to be put back “by hand”)

A unified relativistic treatment of structure formation would automatically solve these issues. When constructing observables (galaxy catalogs, lensing maps etc.), all geometric effects and gauge issues would be treated in a transparent way.

A Case for GR

The Issue of Backreaction

Long standing question: how important is nonlinear evolution of structure for understanding & interpreting the observed "average" cosmological evolution?

References include Ellis 1984, Buchert 2000 & 2008, Schwarz 2002 & 2012, Wetterich 2003, Kolb, Matarrese, Notari & Riotto 2005, Buchert & Ellis 2005, Räsänen 2011

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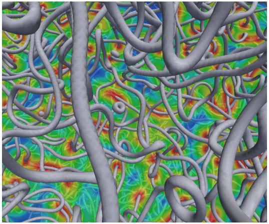
This issue has many facets. Some can be addressed in the Newtonian picture, others require a relativistic treatment ($\rightarrow$ perturbation theory, exact solutions . . .).
A unified relativistic treatment of structure formation would be the logical framework to address the issue in full generality.

A Case for GR

Relativistic Sources of Stress-Energy

GR effects are expected to be important for intrinsically relativistic entities

- Cosmic strings
- Dynamical Dark Energy
- Relativistic particles (neutrinos?)
- ...



credit: Daverio *et al.*

In order to test some of the proposed alternatives/extensions to ΛCDM, general relativistic simulations may be necessary in order to obtain percent accuracy required by future observations (e.g. Euclid)

Metric of perturbed FRW in “longitudinal gauge”

$$ds^2 = -(1 + 2\Psi)\, dt^2 + a^2(t)\left[(1 - 2\Phi)\, \delta_{ij} + h_{ij}\right] dx^i dx^j - 2B_i dx^i dt$$

Gauge condition: $\nabla^i B_i = \nabla^i h_{ij} = \delta^{ij} h_{ij} = 0$

Choice of Variables

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Stress-energy tensor

$$T^\mu_\nu = \bar{T}^\mu_\nu + \delta T^\mu_\nu\,,\ \bar{T}^\mu_\nu = \mathrm{diag}(-\bar{\rho}, \bar{P}, \bar{P}, \bar{P})\,,\ \bar{P} = w\bar{\rho}$$

Fix "background" equation of state for each constituent $\rightarrow$ background scale factor a solves Friedmann's equations for $\bar{T}^\mu_\nu$

Weak Field Approximation

Perturbative approach:

- Metric perturbations Ψ, Φ, ...remain small in cosmological context ($\sim 10^{-5}$) $\rightarrow$ keep only to first order
- Spatial derivatives $\Psi_{,i}$, ...are $\sim v$ ($\sim 10^{-3}$) $\rightarrow$ keep to quadratic order
- Second spatial derivatives $\Delta\Psi$, ...are $\sim \delta$ and therefore non-perturbative

Green & Wald 2012, J.A., Daverio, Durrer & Kunz 2013

The Framework

System of Equations

"$G^0_0 = 8\pi G T^0_0$":

$$\frac{1}{a^2}\left(1+4\Phi\right)\Delta\Phi - 3H\dot{\Phi} - 3H^2\Psi + \frac{3}{2a^2}\left(\nabla\Phi\right)^2 = -4\pi G\delta T^0_0$$

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"$G^i_i - 3G^0_0 - \frac{1}{H}\dot{G}^0_0 = 8\pi G(T^i_i - 3T^0_0 - \frac{1}{H}\dot{T}^0_0)$":

$$\begin{gathered}\left(1+2\Phi-2\Psi\right)\Delta\Psi - \left(\nabla\Psi\right)^2 - \nabla\Psi\nabla\Phi + \\ \frac{1}{H}\partial_t\left[\Delta\Phi + 4\Phi\Delta\Phi + \frac{3}{2}\left(\nabla\Phi\right)^2\right] = \\ 4\pi G\frac{a}{H}\left[\delta T^i_{0,i} - \delta T^i_0\left(3\Phi_{,i} - \Psi_{,i} + a\dot{B}_i\right)\right. \\ \left. -a\dot{\Phi}\left(3\delta T^0_0 - \delta T^i_i\right) - \frac{a}{2}\delta^{ik}\dot{h}_{jk}\delta T^j_i\right]\end{gathered}$$

System of Equations (cont.)

“$G^0_i = 8\pi G T^0_i$”:

$$-\frac{4}{a^2}\Delta B_i - \frac{1}{a}\dot{\Phi}_{,i} - \frac{H}{a}\Psi_{,i} = 4\pi G \delta T^0_i$$

The Framework
System of Equations (cont.)

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$$-\frac{4}{a^2}\Delta B_i - \frac{1}{a}\dot{\Phi}_{,i} - \frac{H}{a}\Psi_{,i} = 4\pi G \delta T^0_i$$

"$G^i_j - \frac{1}{3}\delta^i_j G^k_k = 8\pi G(T^i_j - \frac{1}{3}\delta^i_j T^k_k)$":

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\Delta h_{ij} + \frac{1}{a}\left(\dot{B}_{(i,j)} + 2HB_{(i,j)}\right)$$
$$+\frac{1}{a}\text{ “traceless part }\left[\left(1+4\Phi\right)\Phi_{,ij} - \left(1+2\Phi-2\Psi\right)\Psi_{,ij}\right.$$
$$\left.+\Psi_{,i}\Psi_{,j} - 2\Phi_{(,i}\Psi_{,j)} + 3\Phi_{,i}\Phi_{,j}\right]\text{”} = 8\pi G\left(\delta_{ik}\delta T^k_j - \frac{1}{3}\delta_{ij}\delta T^k_k\right)$$

System of Equations (cont.)

In order to close the system of equations, one needs evolution equations for all sources of stress-energy.

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Geodesic equation for nonrelativistic massive particles

$$\dot{v}^i + H v^i + \delta^{ij} \left(\frac{1}{a} \Psi_{,j} - \dot{B}_j - H B_j + \frac{2}{a} B_{[j,k]} v^k \right) = 0$$

determines the evolution of the particle ensemble and therefore the evolution of the full T^μ_ν of CDM.

Φ: parabolic equation (diffusion type)

- Explicit scheme too inefficient (Courant condition!)
- First-order (in time) implicit scheme shows excellent performance in 1D tests
- ADI *(Alternating Direction Implicit)* scheme in 3D should perform well (easy to implement & parallelizable)

Algorithmic Solutions

Φ: parabolic equation (diffusion type)

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Ψ: elliptic equation

- Nonlinear Gauß-Seidel / Multigrid solver shows excellent performance in 1D tests
- Same class of solvers is already used for the Poisson equation in modern Newtonian codes

Algorithmic Solutions (cont.)

B_i: linear elliptic operator

Two possibilities:

- Solve in Fourier space (transverse component can easily be extracted, but incompatible with AMR)
- Use Multigrid solver (gauge condition more difficult to implement)

Algorithmic Solutions (cont.)

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h_{ij}: linear hyperbolic equation (wave equation)

- No conceptual problem, but can be expensive (depending on relevant range of scales)
- h_{ij} does not enter the geodesic equation for massive particles (at our approximation order) $\rightarrow$ expendable, leave for future work

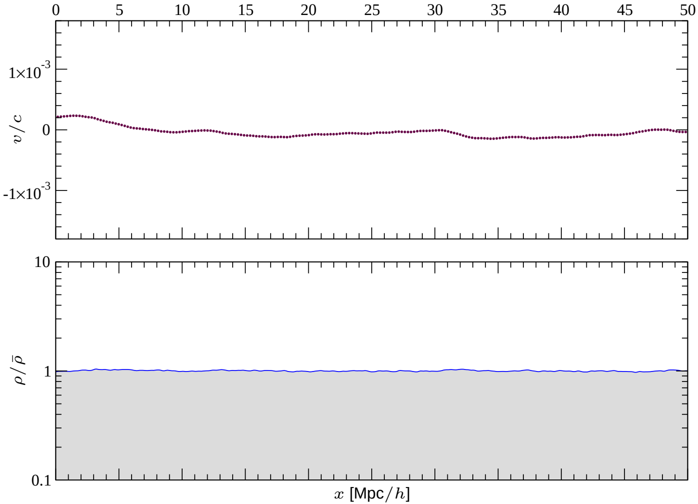
Numerical Results

A Plane-Symmetric Setup

- Restriction to plane-symmetric configuration ($y - z$-plane) trivializes two dimensions $\rightarrow$ high resolution possible with cheap computational requirements (no parallelization)
- No vector & tensor perturbations (by construction)
- 32768 particles, 4096 grid points
- Initial conditions: Gaussian random field obtained from semi-realistic initial power spectrum
- Initialized at $z > 1000$ using linear theory (Zel'dovich approximation)

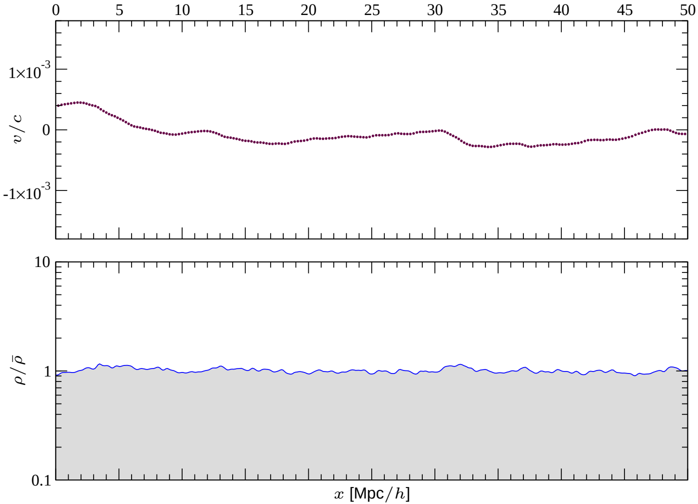
Numerical Results

A Plane-Symmetric Setup



$z = 100$

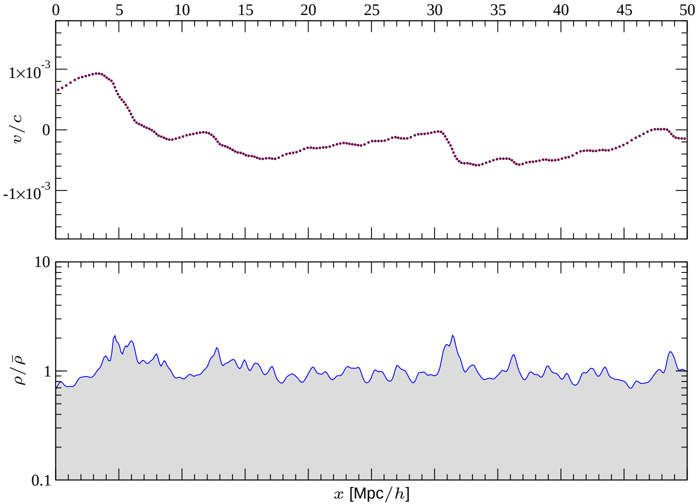
Numerical Results
A Plane-Symmetric Setup



$z = 26$

Numerical Results

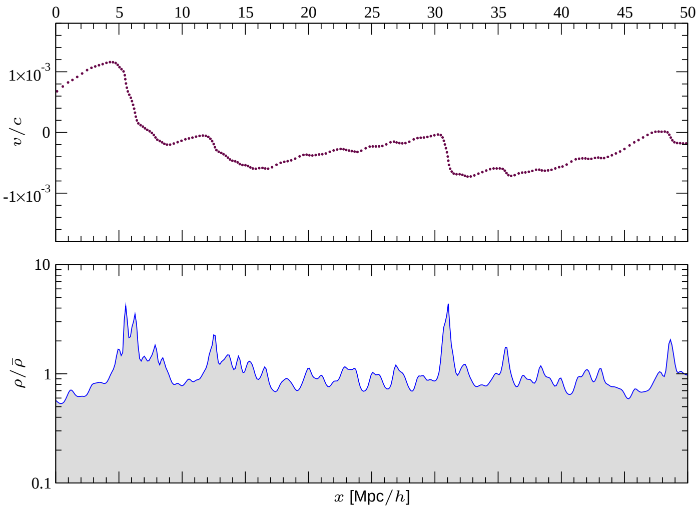
A Plane-Symmetric Setup



$z = 5.3$

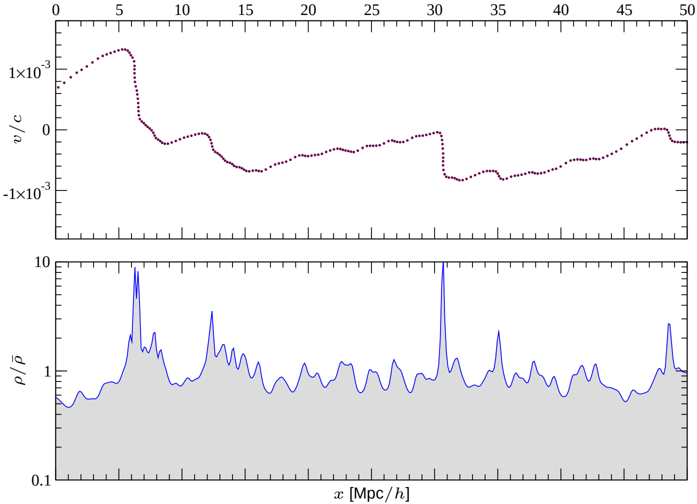
Numerical Results

A Plane-Symmetric Setup



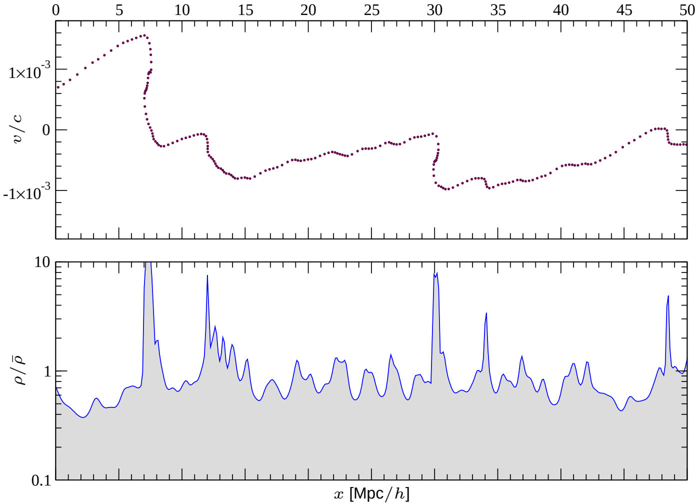
$z = 3.0$

Numerical Results
A Plane-Symmetric Setup



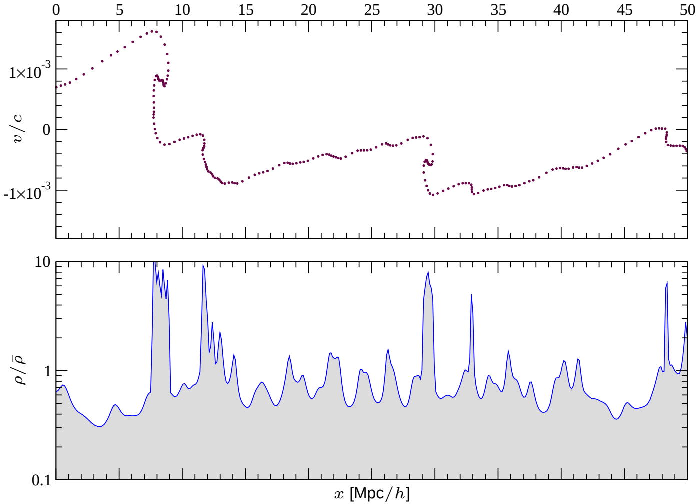
$z = 2.0$

Numerical Results
A Plane-Symmetric Setup



$z = 1.0$

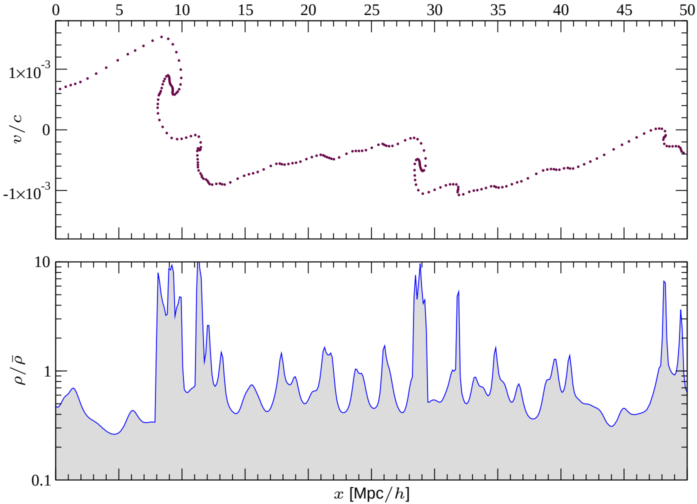
Numerical Results
A Plane-Symmetric Setup



$z = 0.4$

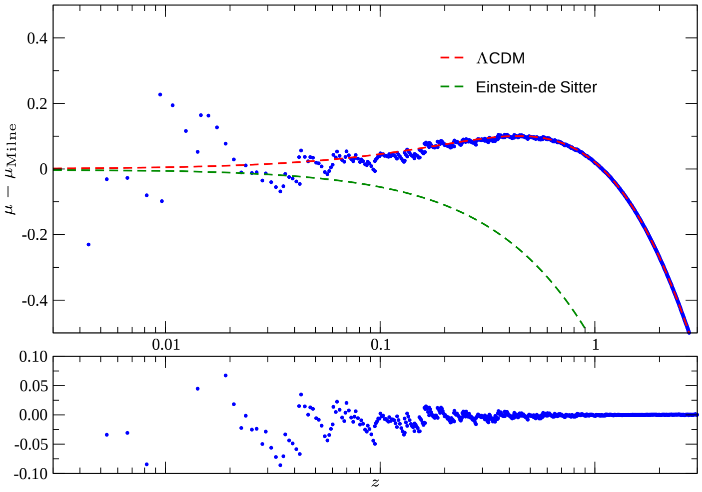
Numerical Results

A Plane-Symmetric Setup



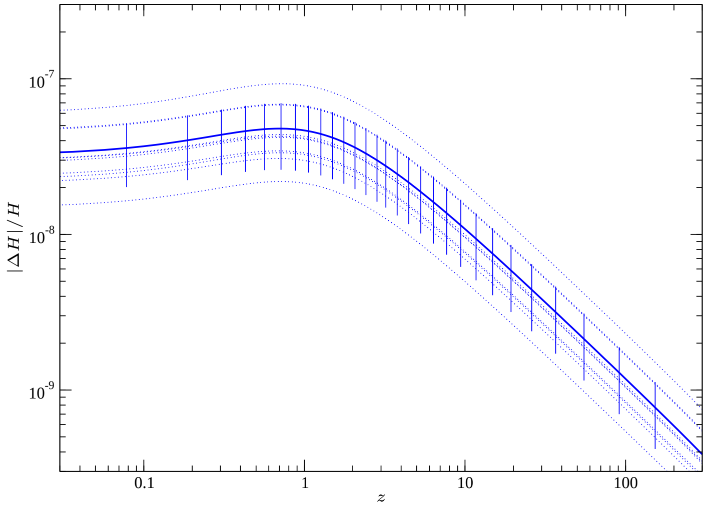
$z = 0.0$

Numerical Results
Luminosity Distance



Numerical Results

Newtonian vs. GR Simulation



Summary

- Cosmological simulations within a **GR framework** are feasible
- A unified relativistic treatment is a clear, logical and transparent way to address the **most general observables** with minimal assumptions about the cosmological model
- Technology is useful for simulations with **relativistic sources** (dynamical DE, cosmic strings, neutrinos) – feasibility depends on ability to model the sources accurately
- For CDM simulations, modifications are computationally relatively inexpensive (but may be unnecessary)
- The issue of **backreaction** can be addressed quantitatively within the non-perturbative regime