Nonlinear correlations and scalings in the halo model

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Halo Model

Objects or events come in clusters, or halos: $N = \sum_{i=1}^{N_h} N_i$

Has been applied in: high energy colliders (hadron multiplicities), photon shot noise statistics (cathodeluminescence radiation from a pulsed electron beam), neurophysiology of vision (nerve impulses), geophysics (earthquake events), hydrology (rainfall totals), species intensity (insect larvae in fields, pine trees in natural forests), diamond deposits, and (doubly) 1990 storm damage in the Fontainebleau forest

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And: galaxy count statistics

B. M. Brown and Ann Cowling, *Biometrika* 85, 427 (1998)

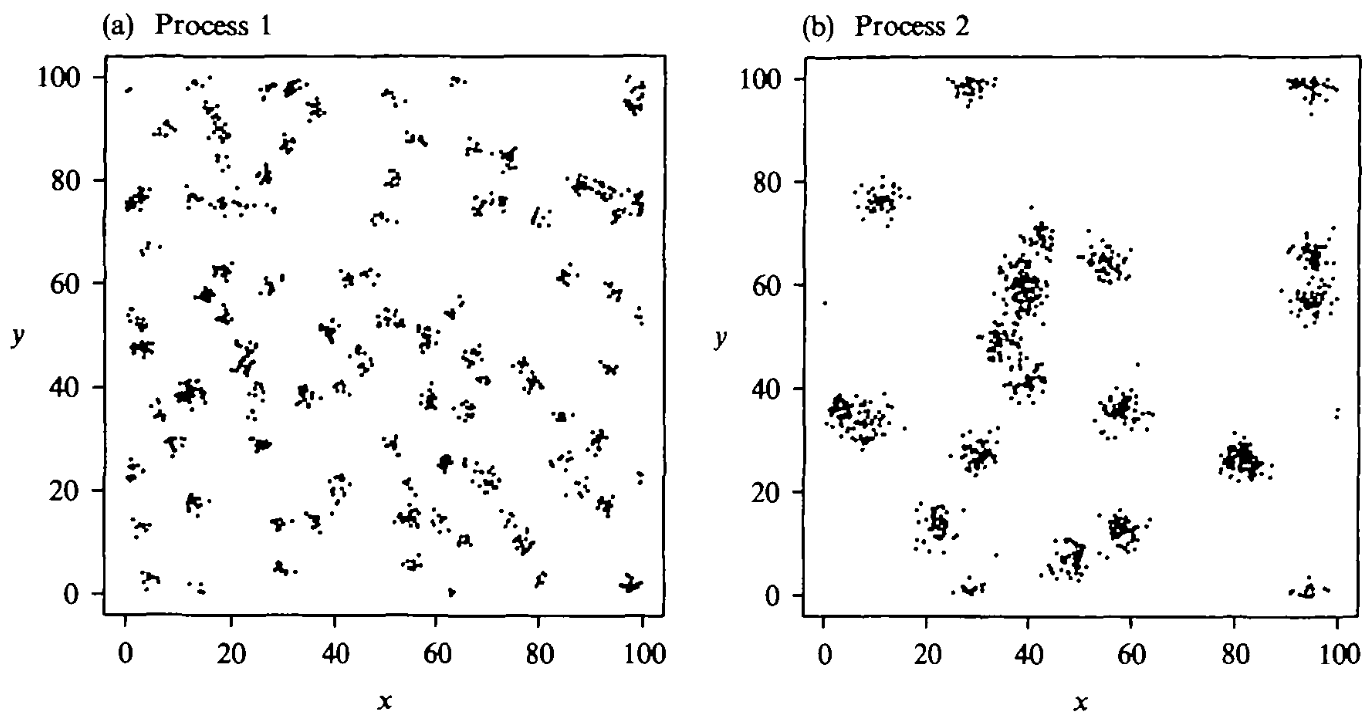

Fig. 1. Realisations of the processes used in the simulation study: (a) process 1, $(\lambda, \mu, \rho) = (0{\cdot}01, 10, 1{\cdot}0)$; (b) process 2, $(\lambda, \mu, \rho) = (0{\cdot}002, 50, 2{\cdot}24)$.

Table 4. *Means (and standard deviation) of estimates of clustering parameters from 250 simulations for Methods 1, 3 and 4, and from 100 simulations for Method 2, with* $(\lambda, \mu, \rho) = (0{\cdot}01, 10, 1)$

$10\sigma = 0{\cdot}80$ $10\sigma = 3{\cdot}19$ $10\sigma = 5{\cdot}59$ $10\sigma = 7{\cdot}98$

Non-conditional and conditional simulation of a spatial point process

J. Caers, J. Gelders, A. Vervoort, and L. Rombouts

Geostatistics Wollongong '96, vol. 1, p.270 (Dordrecht:Kluwer, 1997)

Figure 2. Non-conditional simulation of a diamond deposit

133. Modèle spatial et processus ponctuel.

1331. Processus ponctuels & analyse de la structure spatiale.

Nous avons choisi de représenter les arbres par des points positionnés dans la parcelle**.** Le formalisme des processus ponctuels donne des outils pour analyser, modéliser et simuler la structure spatiale de tels ensembles de points (Ripley, 1981; Diggle, 1983; Cressie, 1993). Les modèles que nous construirons seront précisément des processus ponctuels (figure 2).

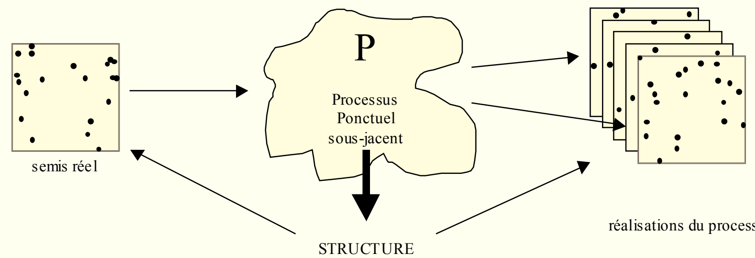

Figure 2 : Un processus ponctuel P est un processus aléatoire (un objet mathématique similaire à une variable aléatoire) dont les réalisations sont des semis de points. Les propriétés du processus définissent des contraintes sur ses réalisations (densité, voisinage, structure...). Un seul processus ponctuel P peut générer une infinité de semis de points, tous différents, mais qui partagent certaines propriétés communes, et en particulier la structure .

Pour analyser la structure spatiale en forêt, et pour ajuster nos modèles, nous utiliserons les fonctions K(r) et L(r) proposées par Ripley[7] (1977) et Besag (1977), qui ont été utilisées récemment pour un grand nombre d'études en forêt (voir une synthèse dans Goreaud, 2000).

1332. Exemple des modèles construits pour les tempêtes de 1967 et 1990 :

L'analyse de la structure spatiale des arbres morts pendant les deux tempêtes de 1967 et 1990 (Goreaud et al., 2000) nous a permis de confirmer le caractère agrégé de la mortalité (voir figure en annexe 3). A partir de ces observations, nous avons pu construire des modèles simples de mortalité par tempête, correspondant à cette structure spatiale, en utilisant des processus ponctuels. Pour la tempête de 1967, nous avons utilisé un processus de Neyman-Scott simple (Tomppo, 1986), et pour la tempête de 1990, nous avons construit un processus ponctuel doublement agrégé (figure 3), en composant deux processus de Neyman-Scott.

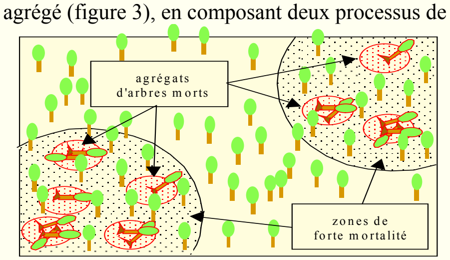

Figure 3 : Principe du modèle de mortalité par tempête doublement agrégé, construit pour rendre compte des dégâts dus à la tempête de 1990.

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[7] Pour un processus ponctuel homogène et isotrope de densité , la fonction K(r) est définie telle que K(r) est l'espérance du nombre de voisins à distance r d'un point quelconque du semis.

Nonlinear correlations and scalings in the halo model

- Halo Model
  Early Halo Model Papers
  "Recent" Halo Model Papers
- Statistics
  Cell Count Moments $\bar{\mu}_k$, $\bar{\xi}_k$, S_k
  Generating Functions (iid)
- Full Halo Model Machinery
  Mass function, bias; Resolved halos, form factors
- Halo model predictions
  Halo $P_N \sim N^{-r}$, $r \approx 2$
  Hierarchical scaling $S_k = \bar{\mu}_k / \bar{\mu}_2^{k-1} \sim \bar{\xi}^{(k-2)\delta}$
  Hierarchical amplitudes $S_k = (2k-3)!!$
- Voids and Scaling
  Hierarchical Scaling
  Scaling in the Halo Model
- (Future) Comparison with Observational Data!
- Conclusions

Early Halo Model Papers

Neyman & Scott 1952 *A Theory of the Spatial Distribution of Galaxies*

Peebles 1974, Peebles & Groth 1975, Peebles 1980 $\rho \sim r^{-\epsilon} \quad \gamma_k = k\,\epsilon - 3 \quad (\epsilon > \frac{3}{2})$

McClelland & Silk 1977 *The correlation function for density perturbations in an expanding universe. II. Nonlinear theory*

McClelland & Silk 1978 *The correlation function for density perturbations in an expanding universe. III. The three-point . . . and higher order correlation functions*

Scherrer & Bertschinger 1980
Statistics of primordial density perturbations from discrete seed masses

Sheth & Saslaw 1994 *Synthesizing the observed distribution of galaxies*

Sheth 1996 *The distribution of counts in cells in the non-linear regime*

Ma & Fry 2000 *Halo Profiles and the Nonlinear Two- and Three-Point Correlation Functions of Cosmological Mass Density*

“Recent” Halo Model Papers

Seljak 2000 *Analytic model for galaxy and dark matter clustering*

Ma & Fry 2000 *Deriving the Nonlinear Cosmological Power Spectrum and Bispectrum from Analytic Dark Matter Halo Profiles and Mass Functions*

Peacock & Smith 2000 *Halo occupation numbers and galaxy bias*

Scoccimarro, Sheth, Hui, & Jain 2001 *How Many Galaxies Fit in a Halo? Constraints on Galaxy Formation Efficiency from Spatial Clustering*

New: specific forms for halo profile, mass function, clustering strength

Applications: Sunyaev-Zel'dovich effect, weak gravitational lensing, redshift space power spectrum, redshift space distortions, redshift space bias and β, nonlinear integrated Sachs-Wolfe effect, pairwise peculiar velocity distribution function, Ly-α forest, patchy reionization and temperature/polarization anisotropies, relic neutrinos and ultra-high-energy cosmic rays, red and blue galaxy populations, luminosity and color dependence of galaxy correlations functions, the history of the baryon budget, baryon acoustic oscillations, synthetic catalogs (covariance matrix), primordial non-Gaussianity

Cell Count Moments

$$\bar{\mu}_k = \frac{1}{\bar{N}^k} \left\langle N^{[k]} \right\rangle = \frac{1}{\bar{M}^k} \left\langle M^k \right\rangle = \frac{1}{V^k} \int d^3 r_1 \cdots d^3 r_k \, \mu(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_k)$$

$$\left\langle N^2 \right\rangle = \bar{N} + \bar{N}^2 \bar{\mu}_2$$

$$\left\langle N^3 \right\rangle = \bar{N} + 3\,\bar{N}^2 \bar{\mu}_2 + \bar{N}^3 \bar{\mu}_3$$

$$\left\langle N^4 \right\rangle = \bar{N} + 7\,\bar{N}^2 \bar{\mu}_2 + 6\,\bar{N}^3 \bar{\mu}_3 + \bar{N}^4 \bar{\mu}_4$$

$$\left\langle N^5 \right\rangle = \bar{N} + 15\,\bar{N}^2 \bar{\mu}_2 + 25\,\bar{N}^3 \bar{\mu}_3 + 10\,\bar{N}^4 \bar{\mu}_4 + \bar{N}^5 \bar{\mu}_5$$

$$\bar{\mu}_2 = 1 + \bar{\xi}_2$$

$$\bar{\mu}_3 = 1 + 3\,\bar{\xi}_2 + \bar{\xi}_3$$

$$\bar{\mu}_4 = 1 + 6\,\bar{\xi}_2 + 3\,\bar{\xi}_2^2 + 4\,\bar{\xi}_3 + \bar{\xi}_4$$

$$\bar{\mu}_5 = 1 + 10\,\bar{\xi}_2 + 15\,\bar{\xi}_2^2 + 10\,\bar{\xi}_3 + 10\,\bar{\xi}_2\,\bar{\xi}_3 + 5\,\bar{\xi}_4 + \bar{\xi}_5$$

Generating Functions

$$M(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\,\bar{\mu}_k\,t^k = \langle\, e^{tx}\,\rangle \qquad K(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\,\bar{\xi}_k\,t^k = \log M(t)$$

$$G(z) = \sum_{N=0}^{\infty} p_N\,z^N = \langle\, z^N\,\rangle$$

$$p_n = \frac{1}{n!}\,\frac{d^n}{dz^n}\,G(z)\Big|_{z=0} \qquad \langle\, N^{[k]}\,\rangle = \frac{d^k}{dz^k}\,G(z)\Big|_{z=1}$$

$$\langle\, N^{[k]}\,\rangle = \frac{d^k}{dt^k}\,M_c(t)\Big|_{t=0} \qquad p_n = \frac{1}{n!}\,\frac{d^n}{dt^n}\,M_c(t)\Big|_{t=-1}$$

$$M_{\text{discrete}}(t) = M_{\text{continuous}}(e^t - 1)$$

$$G(z) = M(z-1)$$

Halo Model Generating Function

Point Cluster Limit (iid) Szapudi & Szalay 1993

$$G(z) = \langle\, z^{N} \,\rangle = \langle\langle\, z^{N_1 + \cdots + N_{N_h}} \,\rangle\rangle$$

$$= \langle\langle\, z^{N_1} \,\rangle \cdots \langle\, z^{N_{N_h}} \,\rangle\rangle = \langle [g_i(z)]^{N_h} \,\rangle = g_h[g_i(z)]$$

$$K(t) = \log[M(t)] = \log[G(t+1)] = \log\big(g_h[g_i(t+1)]\big)$$

$$= \log\big(g_h[M_i(t)]\big) = \log\big(M_h[M_i(t) - 1]\big) = K_h[M_i(t) - 1]$$

Generating Function Results

$$\bar{N} = \bar{N}_h \bar{N}_i$$

$$\bar{\xi}_2 = \bar{\xi}_{2,h} + \frac{\bar{\mu}_{2,i}}{\bar{N}_h}$$

$$\bar{\xi}_3 = \bar{\xi}_{3,h} + \frac{3\,\bar{\mu}_{2,i}\,\bar{\xi}_{2,h}}{\bar{N}_h} + \frac{\bar{\mu}_{3,i}}{\bar{N}_h^2}$$

$$\bar{\xi}_4 = \bar{\xi}_{4,h} + \frac{6\,\bar{\mu}_{2,i}\,\bar{\xi}_{3,h}}{\bar{N}_h} + \frac{(4\,\bar{\mu}_{3,i} + 3\,\bar{\mu}_{2,i}^2)\,\bar{\xi}_{2,h}}{\bar{N}_h^2} + \frac{\bar{\mu}_{4,i}}{\bar{N}_h^3}$$

$$\bar{\xi}_5 = \bar{\xi}_{5,h} + \frac{10\,\bar{\mu}_{2,i}\,\bar{\xi}_{4,h}}{\bar{N}_h} + \frac{(10\,\bar{\mu}_{3,i} + 15\,\bar{\mu}_{2,i}^2)\,\bar{\xi}_{3,h}}{\bar{N}_h^2} + \frac{(10\,\bar{\mu}_{2,i}\bar{\mu}_{3,i} + 5\bar{\mu}_{4,i})\,\bar{\xi}_{2,h}}{\bar{N}_h^3} + \frac{\bar{\mu}_{5,i}}{\bar{N}_h^4}$$

$$
\begin{aligned}
\bar{\xi}_9 = \bar{\xi}_{9,h} &+ \frac{36\,\bar{\mu}_2\,\bar{\xi}_{8,h}}{\bar{N}_h} + \frac{(378\,\bar{\mu}_2^2 + 84\,\bar{\mu}_3)\,\bar{\xi}_{7,h}}{\bar{N}_h^2} + \frac{(1260\,\bar{\mu}_2^3 + 1260\,\bar{\mu}_2\bar{\mu}_3 + 126\,\bar{\mu}_4)\,\bar{\xi}_{6,h}}{\bar{N}_h^3} \\
&+ \frac{(945\,\bar{\mu}_2^{\,4} + 3780\,\bar{\mu}_2^{\,2}\bar{\mu}_3 + 840\,\bar{\mu}_3^{\,2} + 1260\,\bar{\mu}_2\bar{\mu}_3 + 126\,\bar{\mu}_5)\,\bar{\xi}_{5,h}}{\bar{N}_h^4} \\
&+ \frac{(1260\,\bar{\mu}_2^3\bar{\mu}_3 + 2520\,\bar{\mu}_2\bar{\mu}_3^{\,2} + 1890\,\bar{\mu}_2^{\,2}\bar{\mu}_4 + 1260\,\bar{\mu}_3\bar{\mu}_4 + 756\,\bar{\mu}_2\bar{\mu}_5 + 84\,\bar{\mu}_6)\,\bar{\xi}_{4,h}}{\bar{N}^5} \\
&+ \frac{(280\,\bar{\mu}_3^{\,3} + 1260\,\bar{\mu}_2\bar{\mu}_3\bar{\mu}_4 + 315\,\bar{\mu}_4^{\,2} + 378\,\bar{\mu}_2^{\,2}\bar{\mu}_5 + 504\,\bar{\mu}_3\bar{\mu}_5 + 252\,\bar{\mu}_2\bar{\mu}_6 + 36\,\bar{\mu}_7)\,\bar{\xi}_{3,h}}{\bar{N}_h^6} \\
&+ \frac{(126\,\bar{\mu}_4\bar{\mu}_5 + 84\,\bar{\mu}_3\bar{\mu}_6 + 36\,\bar{\mu}_2\bar{\mu}_7 + 9\,\bar{\mu}_8)\,\bar{\xi}_{2,h}}{\bar{N}^7} + \frac{\bar{\mu}_{9,i}}{\bar{N}_h^8}
\end{aligned}
$$

Halo Model Inputs

$$\frac{\rho(r)}{\bar{\rho}} = A\, u(\frac{r}{r_s})$$ (NFW, Moore et al., . . .)

$$\frac{\mathrm{d}n}{\mathrm{d}m} = \frac{\mathrm{d}\ln\nu}{\mathrm{d}\ln m}\,\frac{\bar{n}}{m^2}\;\nu\, f(\nu)$$

$$\nu\, f(\nu) = 2A\left[1 + (q\nu^2)^{-p}\right]\sqrt{\frac{q\nu^2}{2\pi}}\; e^{-q\nu^2/2}$$ (PS, ST, . . .)

$$b = 1 + \frac{1}{\delta_c}(q\nu^2 - 1) + \frac{2p}{\delta_c}\,\frac{1}{[1 + (q\nu^2)^p]}$$ (MJW, ST, . . .)

One-Halo Term

$$G(z) = \langle\, z^N \,\rangle = \langle\langle\langle\, z^{N_1 + \cdots + N_{N_h}} \,\rangle_{N_i} \,\rangle_{N_h} \,\rangle_m$$

$$\bar{\xi}_k^{1h} = \frac{\langle\, N_i^{[k]} \,\rangle^{1h}}{\langle\, N_i \,\rangle^k} \to \frac{\int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)\, V \,\langle\, N^{[k]}(m) \,\rangle}{\left[\int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)\, V \,\langle\, N(m) \,\rangle\right]^k}$$

$$\int \mathrm{d}m \, \frac{\mathrm{d}n}{\mathrm{d}m} \, V \,\langle\, N(m) \,\rangle = \bar{n}_h V \,\langle\, N \,\rangle = \bar{N}_h \bar{N}_i = \bar{N}$$

$$\int \mathrm{d}m \, \frac{\mathrm{d}n}{\mathrm{d}m} \, V \,\langle\, N^{[k]}(m) \,\rangle = \bar{n}_h V \,\langle\, N^{[k]} \,\rangle = \bar{N}_h \bar{N}_i^k \, \bar{\mu}_{k,i}$$

$$\bar{\mu}_{k,i} = \frac{\int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)\,\langle\, N^{[k]}(m) \,\rangle \;/\; \int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)}{\left[\int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)\,\langle\, N(m) \,\rangle \;/\; \int \mathrm{d}m\, (\mathrm{d}n/\mathrm{d}m)\right]^k}$$

Multiple-Halo Terms

$$\bar{N}^k \bar{\xi}_k^{kh} = \int \mathrm{d}m_1 \, \frac{\mathrm{d}n}{\mathrm{d}m_1} \, \langle \, N(m_1) \, \rangle \, b(m_1) \cdots \int \mathrm{d}m_k \, \frac{\mathrm{d}n}{\mathrm{d}m_k} \, \langle \, N(m_k) \, \rangle \, b(m_k)$$

$$\times \int d^3 r_1' \cdots d^3 r_k' \, F(r_1') \cdots F(r_k') \, \xi_{k,\rho}(\boldsymbol{r}_1', \ldots, \boldsymbol{r}_k') = \bar{N}^k \, \frac{\bar{b}^k}{\bar{b}_h^k} \, \bar{\xi}_{k,h}$$

$$\bar{b} = \frac{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, \langle \, N(m) \, \rangle \, b(m)}{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, \langle \, N(m) \, \rangle} \qquad \bar{b}_h = \frac{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, b(m)}{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m)}$$

$$\bar{\xi}_{k_1+k_2}^{2h} = \frac{s \, k!}{k_1! \, k_2!} \, \bar{\mu}_{k_1} \bar{\mu}_{k_2} \, \frac{\bar{b}_{k_1} \bar{b}_{k_2}}{\bar{b}_h^2} \, \bar{\xi}_{2,h}$$

$$\bar{b}_k = \frac{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, \langle \, N^{[k]}(m) \, \rangle \, b(m)}{\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, \langle \, N^{[k]}(m) \, \rangle}$$

Resolved Halos

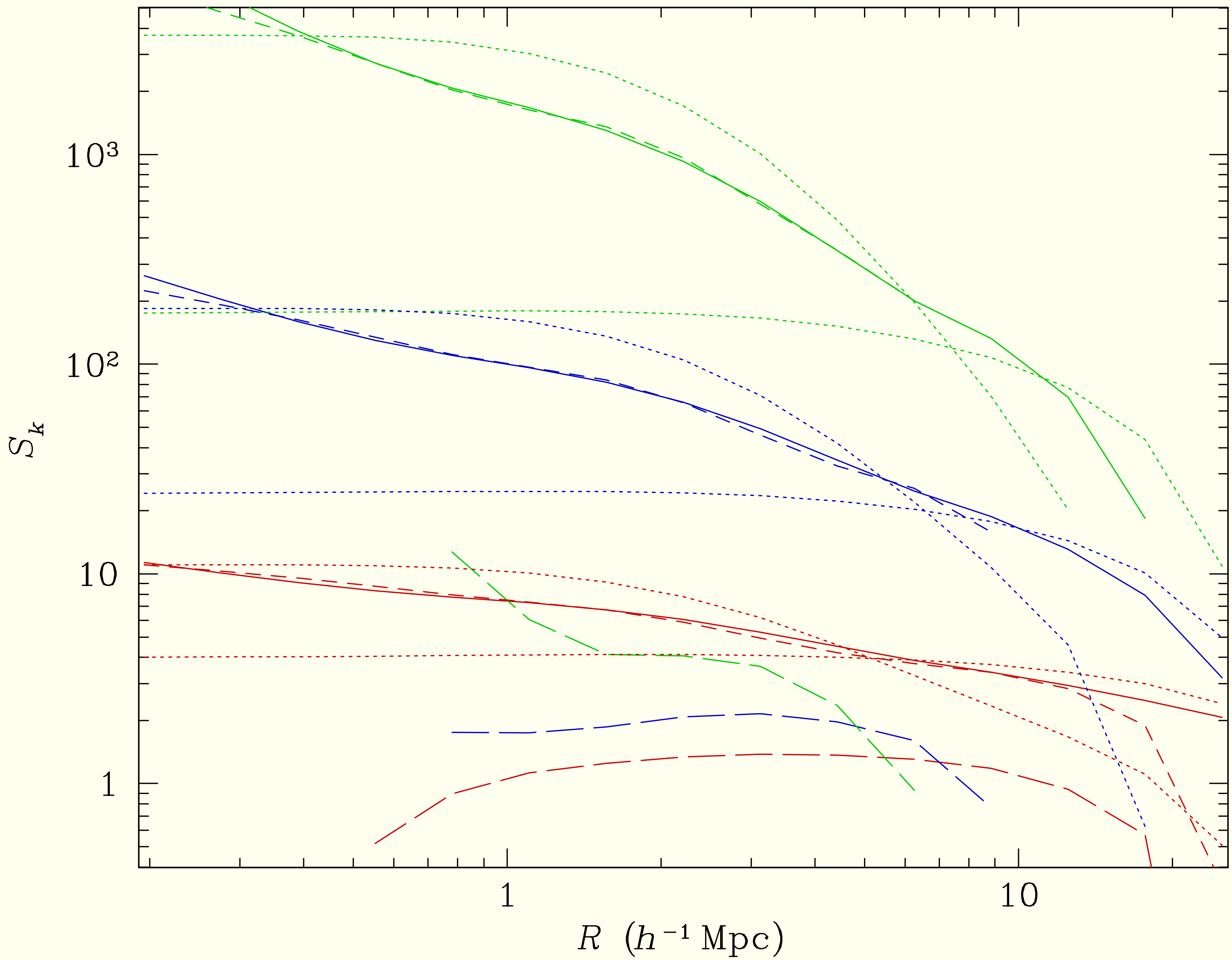
$$\langle\, N'(N'-1)\,\rangle = \langle\, \sum_{i=1}^{N} N_i \,\big(\sum_{j=1}^{N} N_j - 1\big)\,\rangle$$

$$= \langle\, \sum_{i\neq j} N_i N_j + \sum_{i=1}^{N} N_i^2 - \sum_{i=1}^{N} N_i \,\rangle = \langle\, \sum_{i\neq j} N_i N_j \,\rangle = N(N-1)\,\langle\, p^2 \,\rangle$$

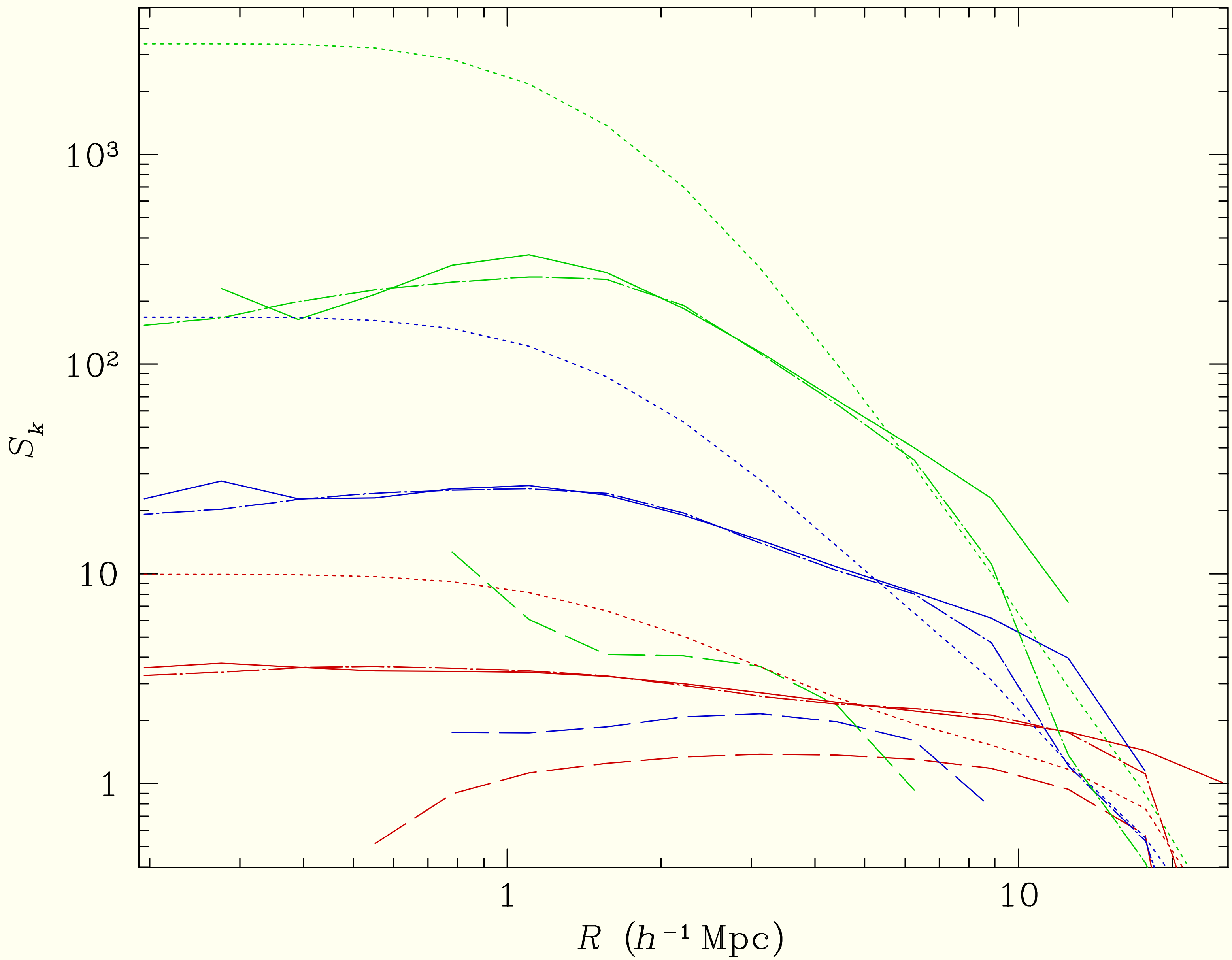
$$p = F(r') = \int_0^R \mathrm{d}^3 r\; u\Big(\frac{\boldsymbol{r}-\boldsymbol{r}'}{r_s}\Big)$$

$$\langle\, p^k \,\rangle = \langle\, F^k \,\rangle = \frac{1}{V}\int_0^\infty d^3 r'\,\big[F(\boldsymbol{r}')\big]^k$$

$$\bar{\mu}_k = \frac{\int \mathrm{d}m\,(\mathrm{d}n/\mathrm{d}m)\,\langle\, N^{[k]}\,\rangle\,\langle\, F^k\,\rangle\,/\,\bar{n}_h}{\big[\int \mathrm{d}m\,(\mathrm{d}n/\mathrm{d}m)\,\langle\, N(m)\,\rangle\,/\,\bar{n}_h\big]^k}$$



10^3
10^2
10
1
S_k
1
10
R (h^{-1} Mpc)



10^3
10^2
10
1
S_k
1
10
R (h^{-1} Mpc)

Statistics of Number

Hydrodynamic simulations, semi-analytic models: mean count *almost* linear

$$\langle\, N(m)\,\rangle \sim m^{\beta}, \qquad \beta < 1$$

Higher order moments typically sub-Poisson,

$$\langle\, N(N-1)\,\rangle < \langle\, N\,\rangle^2$$

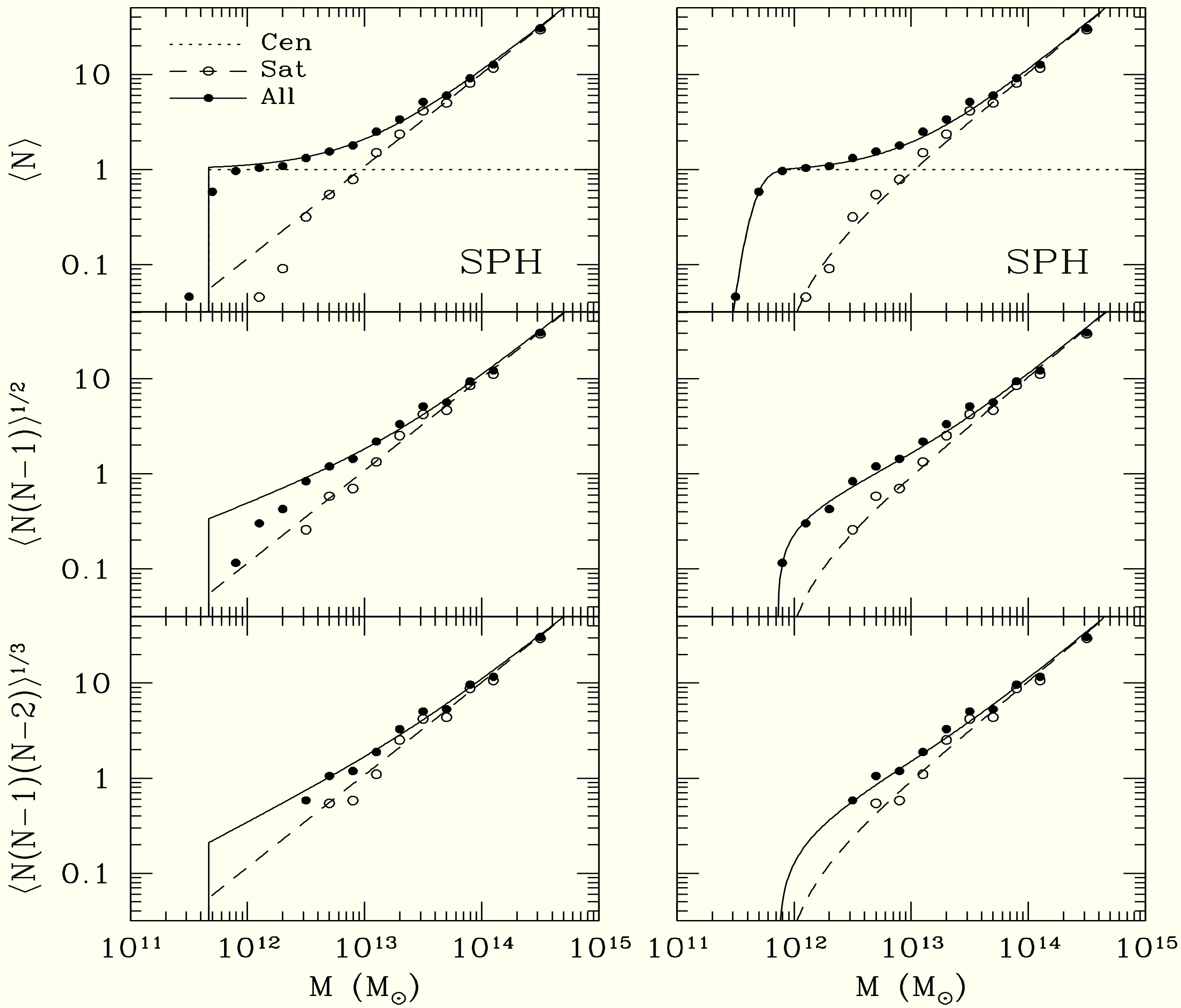
Useful model: central object plus a Poisson distribution of satellites:

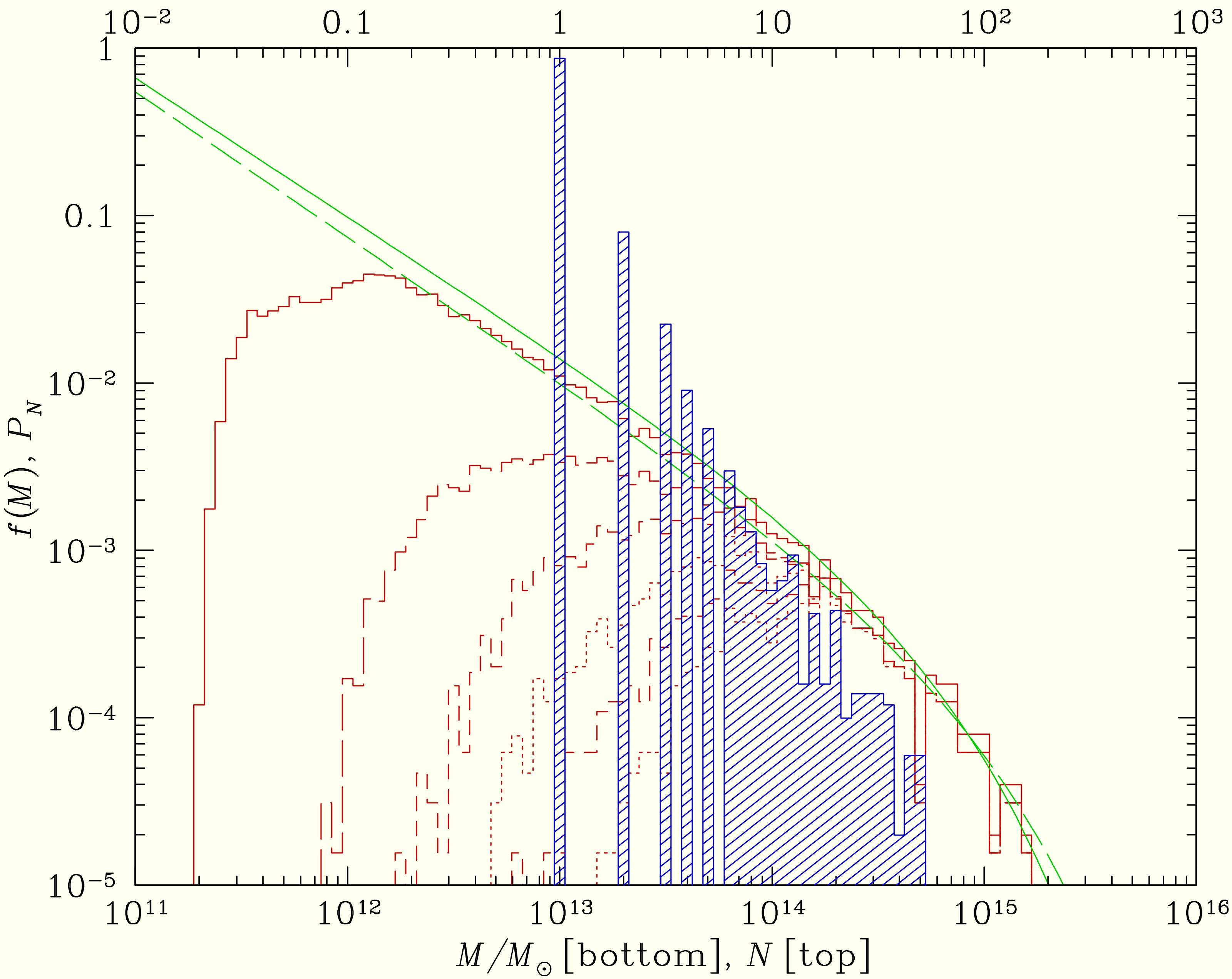
$$N = N_c + N_s$$

$$\langle\, N^{[k]}\,\rangle = \langle\, N_s^{[k]}\,\rangle + k\,\langle\, N_c\,\rangle\,\langle\, N_s^{[k-1]}\,\rangle$$

Theoretical Models of the Halo Occupation Distribution: Separating Central and Satellite Galaxies

Z. Zheng et al., 2005, ApJ, 633, 791





10^{-2}
0.1
1
10
10^2
10^3
1
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$f(M)$, P_N
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
$M/M_\odot$ [bottom], N [top]

Number Distribution P_N

$$\bar{N}(m) \sim m^{\beta} \qquad \nu \sim m^{(3+n)/6}$$

$$P_N \propto \int \mathrm{d}m \, \frac{\mathrm{d}n}{\mathrm{d}m} \, \frac{1}{N!} \, \bar{N}^N e^{-\bar{N}} \propto \frac{1}{N!} \int \mathrm{d}m \, \frac{m^{(3+n)p'/6}}{m^2} \, m^{N\beta} \, e^{-m^{\beta}}$$

$$\propto \frac{[N + (3+n)p'/6\beta - 1 - 1/\beta]!}{N!} \sim N^{-r}$$

$$r = 1 + \frac{1}{\beta} - \frac{(3+n)p'}{6\beta} \approx 2$$

Correlation Function Scalings

$$P(k) \sim k^n, \qquad \nu \sim m^{(3+n)/6}, \qquad c(m) \sim m^{-(3+n)/6},$$

$$\langle\, N \,\rangle \sim m^{\beta}, \qquad \langle\, N^{[k]} \,\rangle \sim m^{k\beta}, \qquad \frac{\mathrm{d}n}{\mathrm{d}m} \sim \frac{\nu^{p'}}{m^2} \quad (p' = 1 - 2p)$$

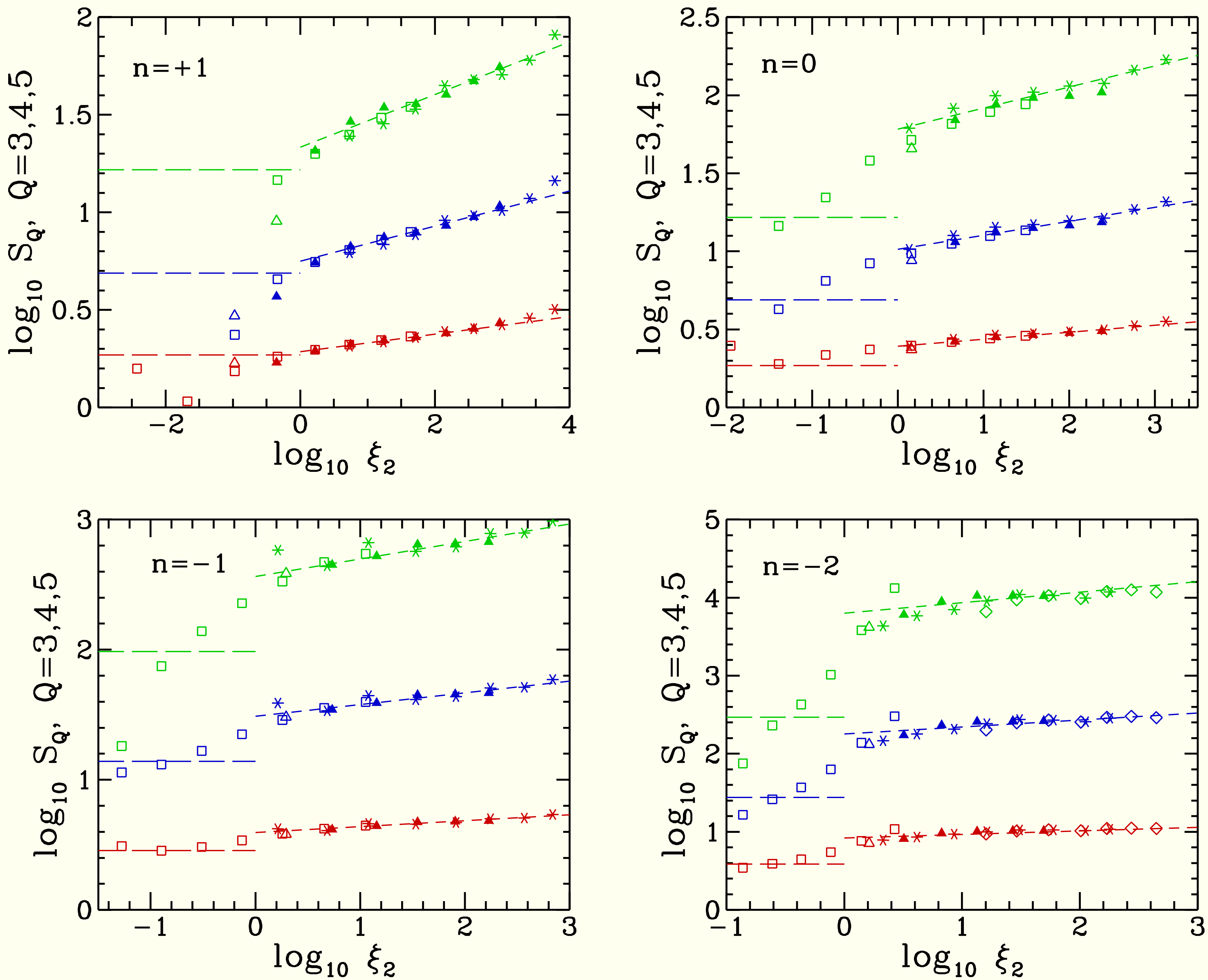
$$\bar{\xi}_k \sim R^{-\gamma_k}, \qquad \gamma_k = (k-1)\,\frac{3(5-2\beta+n)}{5+n} + \frac{6(1-\beta)}{5+n} - \frac{(3+n)p'}{5+n}$$

$$S_k \sim R^{(k-2)\Delta\gamma}, \qquad \Delta\gamma = \frac{(3+n)p' - 6(1-\beta)}{5+n}$$

$$S_k \sim \bar{\xi}^{(k-2)\delta}, \qquad \delta = \frac{p' - 6(1-\beta)/(3+n)}{3 - p' + 12(1-\beta)/(3+n)}$$

Self-similarity and scaling behavior of scale-free gravitational clustering

Colombi, Bouchet, & Hernquist 1996



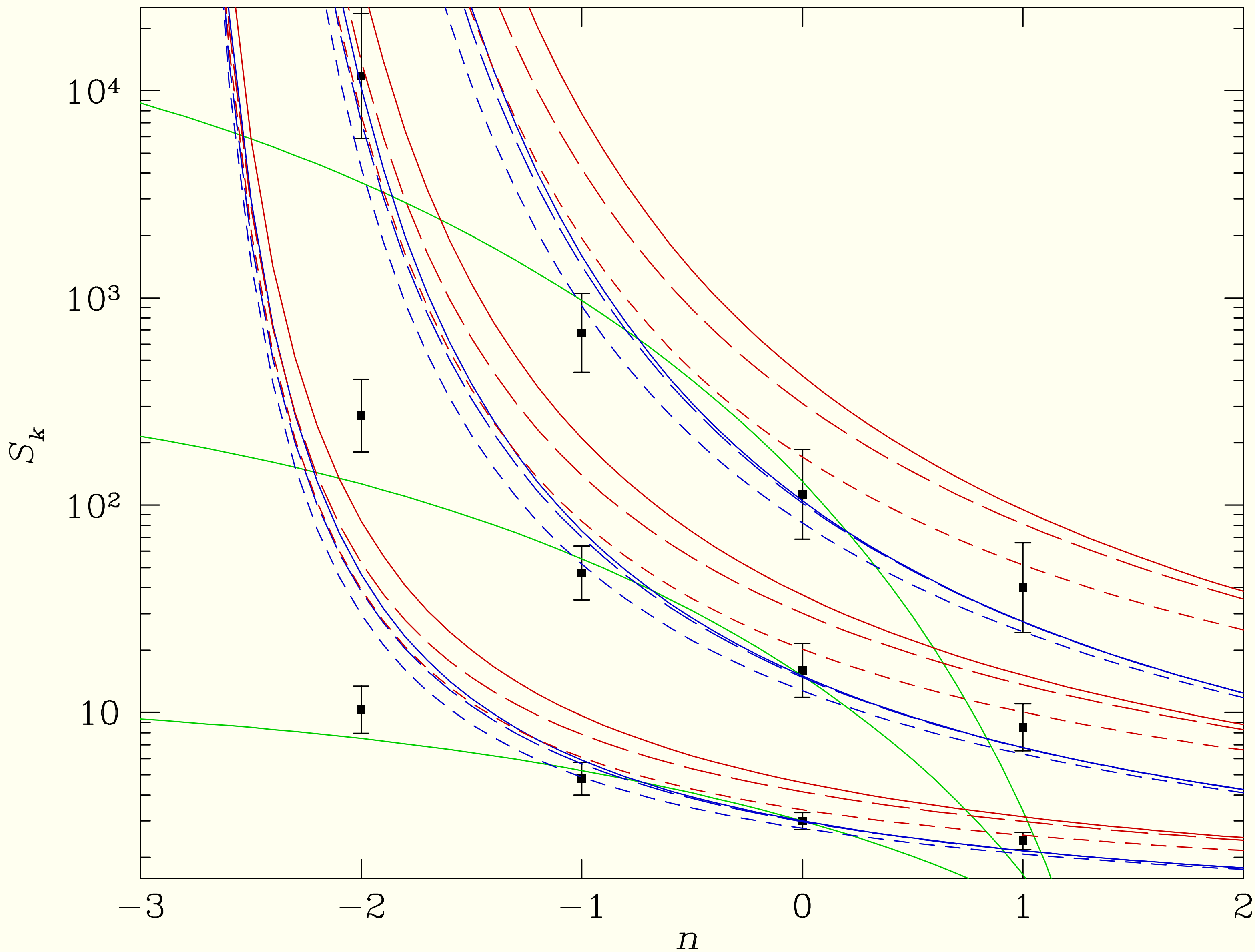
Amplitudes from First Principles

$$\bar{\mu}_k = \frac{\left[\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, m^k \,\right] \left[\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \,\right]^{k-1}}{\left[\int \mathrm{d}m \, (\mathrm{d}n/\mathrm{d}m) \, m \,\right]^k},$$

$$S_k = \frac{\bar{\mu}_k}{\bar{\mu}_2^{k-1}} = \frac{\left[\int \mathrm{d}\nu \, f(\nu) \, m^{k-1}\right] \left[\int \mathrm{d}\nu \, f(\nu)\right]^{k-2}}{\left[\int \mathrm{d}\nu \, f(\nu) \, m\right]^{k-1}}$$

$$S_k = \frac{I(k) \, \left[I(1)\right]^{k-2}}{\left[I(2)\right]^{k-1}},$$

$$I(k) = \Gamma\left[\frac{3(k-1)}{3+n} + \frac{1}{2}\right] + 2^{-p} \, \Gamma\left[\frac{3(k-1)}{3+n} + \frac{1}{2} - p\right],$$



10^4
10^3
10^2
10
S_k
−3
−2
−1
0
1
2
n

Voids and Hierarchical Scaling

Fall, Geller, Jones & White 1976; White 1979; Fry 1984

$$P_0 = \exp\Big[\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\, \bar{N}^n \bar{\xi}_n\Big]$$

Hierarchical $\bar{\xi}_n = S_n\, \bar{\xi}^{n-1}$ Sharp 1981; Schaeffer 1984; Fry 1986; Balian & Schaeffer 1989

$$P_0 = \exp\Big[-\bar{N} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!}\, S_n\, (\bar{N}\bar{\xi})^{n-1}\Big]$$

$$-\frac{\log P_0}{\bar{N}} = \chi(\bar{N}\bar{\xi}), \qquad \bar{N}\bar{\xi} = \frac{\langle\, N^2 \,\rangle - \langle\, N \,\rangle^2 - \langle\, N \,\rangle}{\langle\, N \,\rangle}$$

Some Void Models

Minimal $S_n = 1$

$$\chi = \frac{1}{\bar{N}\bar{\xi}}(1 - e^{-\bar{N}\bar{\xi}})$$

Negative Binomial $S_n = (n-1)!$ Carruthers & Shih 1983; Carruthers & Minh 1983

$$\chi = \frac{1}{\bar{N}\bar{\xi}} \log(1 + \bar{N}\bar{\xi})$$

Thermodynamic $S_n = (2n-3)!!$ Saslaw & Hamilton 1984

$$\chi = \frac{1}{\sqrt{1 + \bar{N}\bar{\xi}}}$$

Lognormal $S_n = n^{n-2}$ $(Q_n = 1)$ Schaeffer 1984; Bernardeau and Schaeffer 1999

$$\bar{N}\bar{\xi} = \tau\, e^{\tau}, \qquad \chi = \left(1 + \tfrac{1}{2}\,\tau\right) e^{-\tau}$$

Gravitational Instability Bernardeau 1992

Observations

Maurogordato & Lachieze-Rey et al., 1987–1992
CfA, SSRS, Abell/PGH/Tully clusters, scaling with luminosity/depth, . . .

Fry, Giovanelli, Haynes, Melott, & Scherrer 1989 Perseus-Pisces

Vogeley, Geller, & Huchra 1991 *Void statistics of the CfA redshift survey*

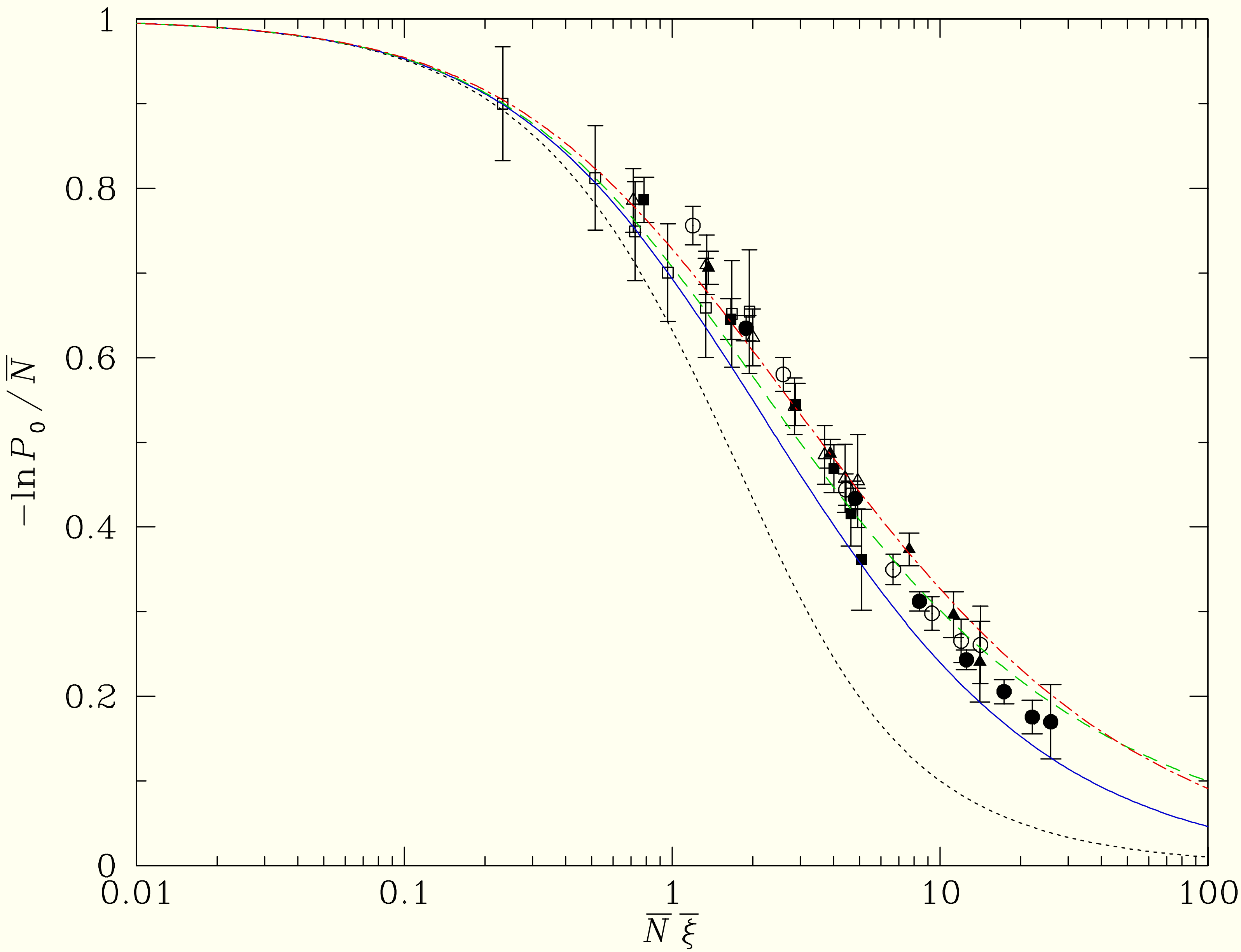
Bouchet, Strauss, Davis, Fisher, Yahil, & Huchra 1993
Counts Distribution in the $1.2\,\mathrm{Jy}$ $\mathbf{IRAS}$ *Galaxy Survey*

Croton et al. 2004 *The 2dF Galaxy Redshift Survey: voids and hierarchical scaling models*

Croton, Norberg, Gaztañaga, & Baugh 2007 *Statistical analysis of galaxy surveys – III. The non-linear clustering of red and blue galaxies in the 2dFGRS*

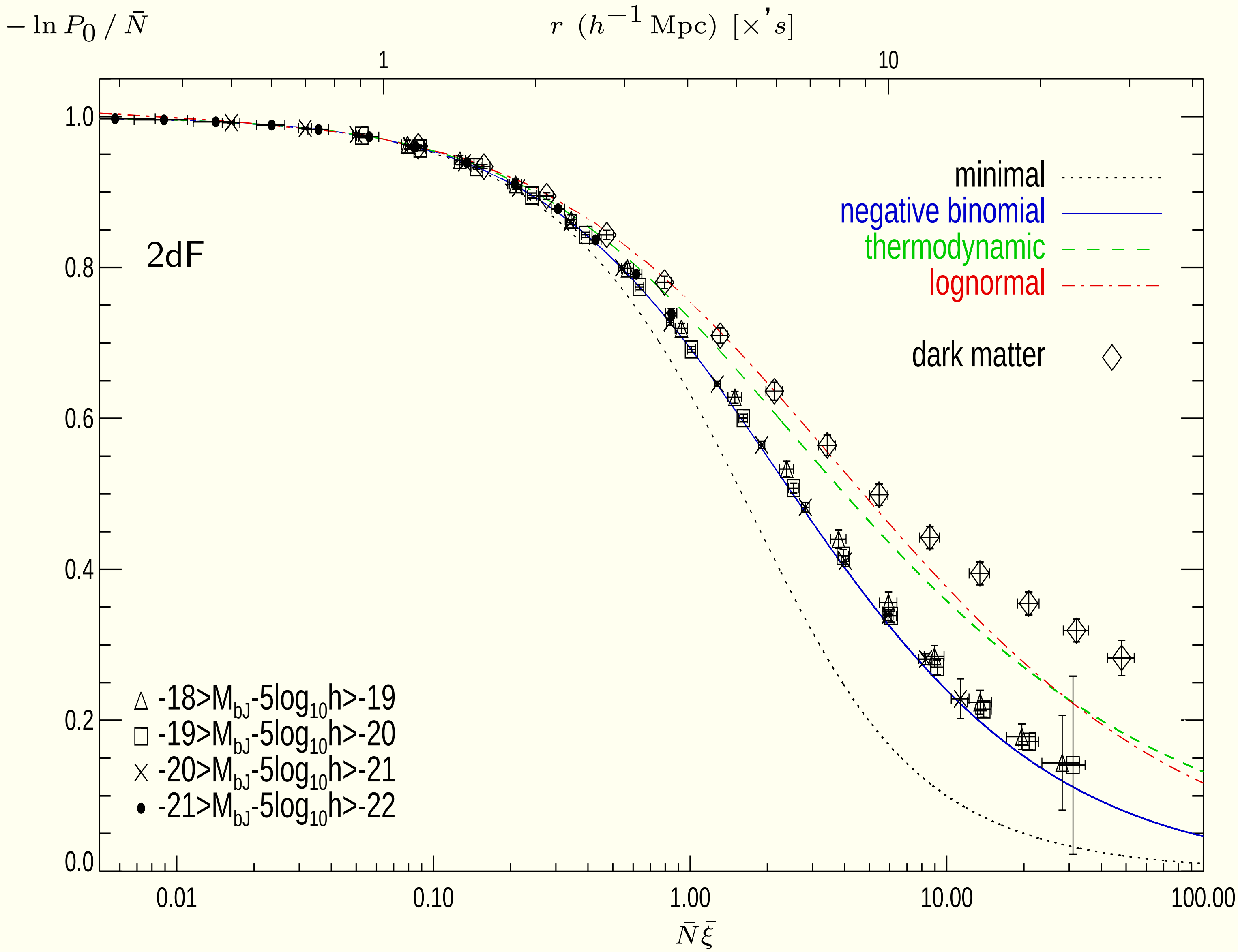
Tinker, Conroy, Norberg, Patiri, Weinberg, & Warren 2008 *Void Statistics in Large Galaxy Redshift Surveys: Does Halo Occupation of Field Galaxies Depend on Environment?*

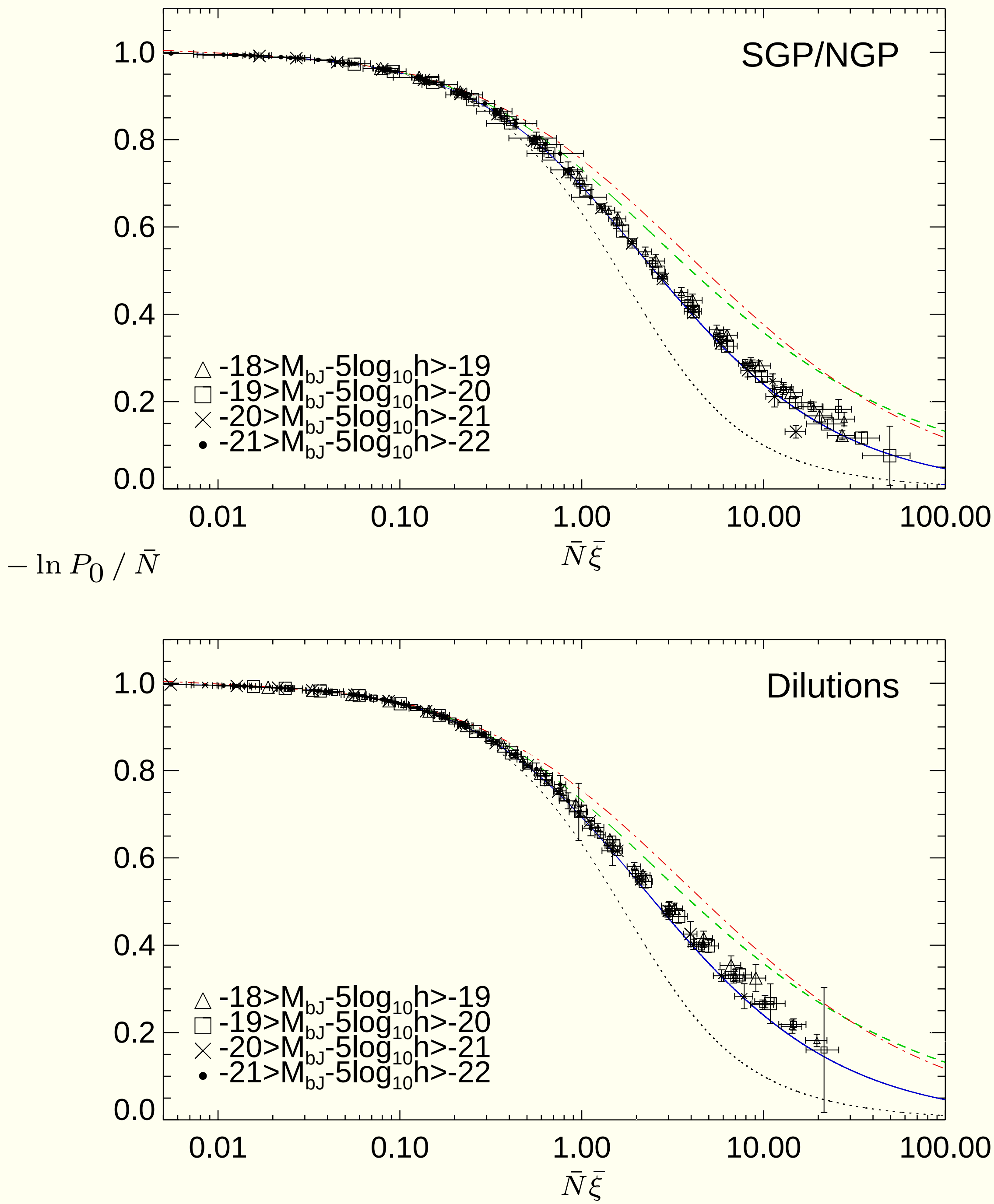
Fry, Giovanelli, Haynes, Melott, & Scherrer 1989



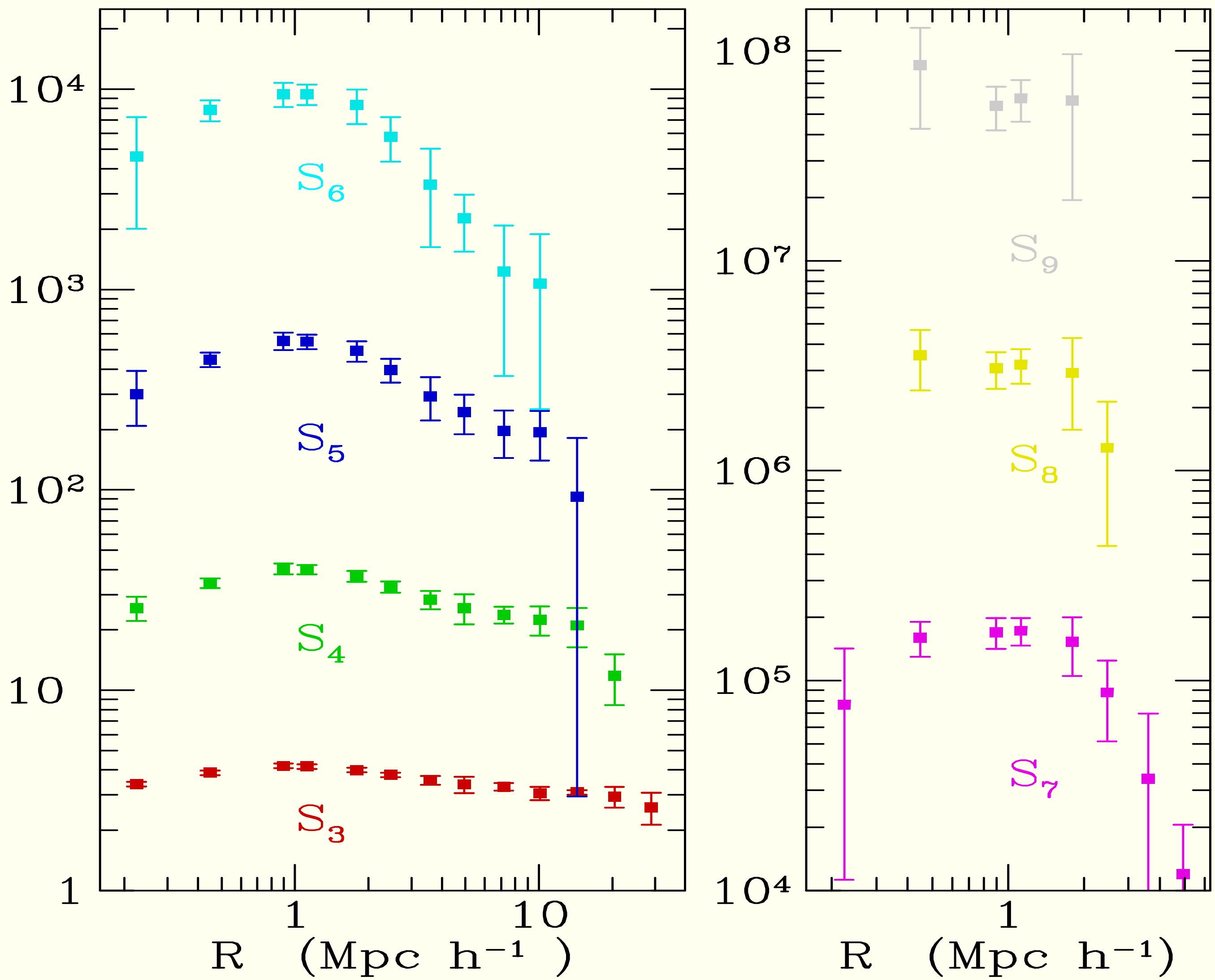
The 2dF Galaxy Redshift Survey: voids and hierarchical scaling models

Croton et al. 2004, MNRAS 352, 828





SGP/NGP
-18>M_bJ-5log_10h>-19
-19>M_bJ-5log_10h>-20
-20>M_bJ-5log_10h>-21
-21>M_bJ-5log_10h>-22
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.10
1.00
10.00
100.00
$\bar{N}\bar{\xi}$
$-\ln P_0 / \bar{N}$
Dilutions
-18>M_bJ-5log_10h>-19
-19>M_bJ-5log_10h>-20
-20>M_bJ-5log_10h>-21
-21>M_bJ-5log_10h>-22
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.10
1.00
10.00
100.00
$\bar{N}\bar{\xi}$



E. Gaztanaga 1994, MNRAS, 269, 915, Figure 5

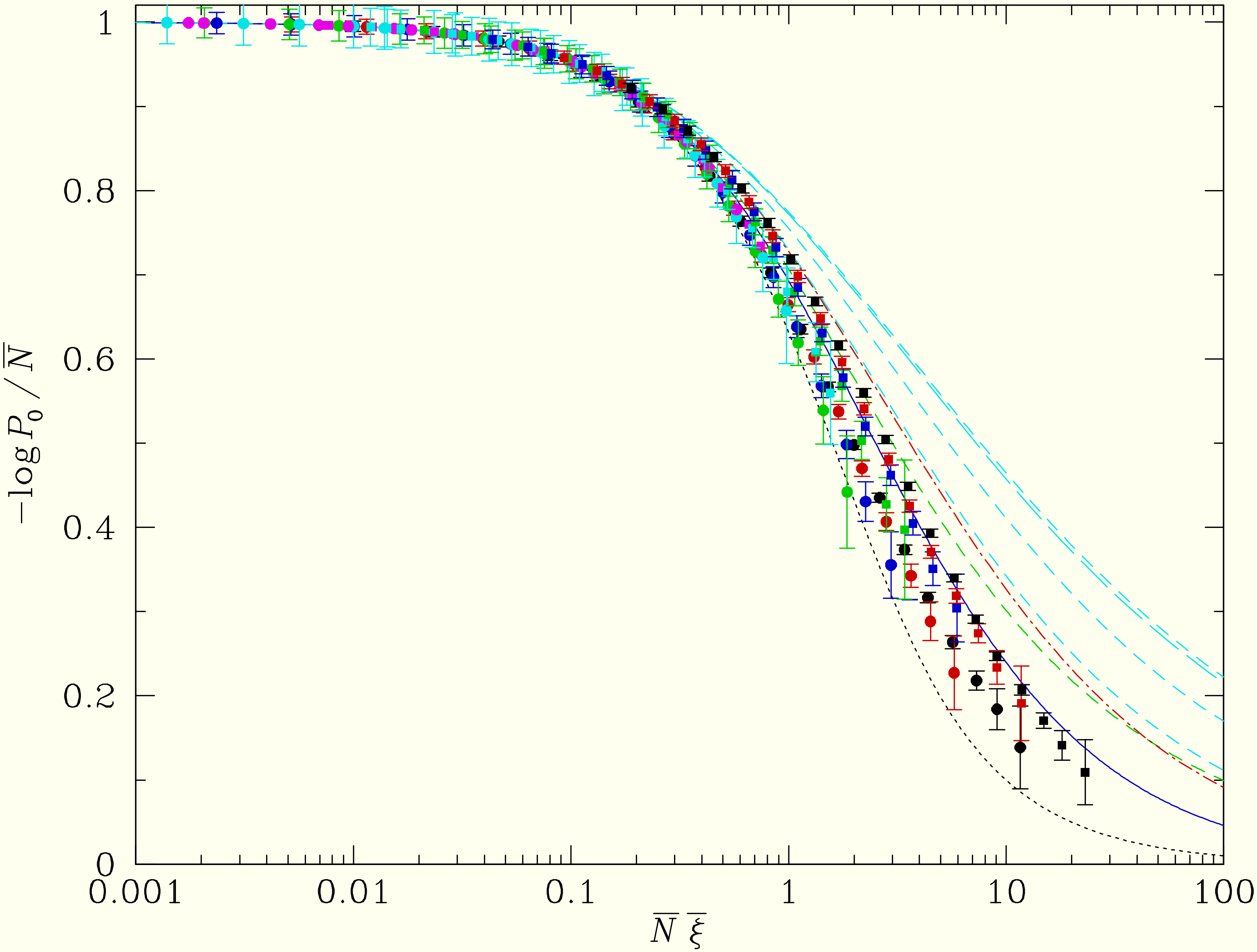
Void Scaling in the Halo Model

$$\bar{N}_g \bar{\xi}_g \; = \frac{\bar{N}_g}{\bar{N}_h} \, \left(\bar{N}_h \bar{\xi}_h + \bar{\mu}_2 \right)$$

$$\bar{N}_g^2 \bar{\xi}_{3,g} = \frac{\bar{N}_g^2}{\bar{N}_h^2} \left[S_{3,h} (\bar{N}_h \bar{\xi}_h)^2 + 3\bar{\mu}_2 \, \bar{N}_h \bar{\xi}_h + \bar{\mu}_3 \right],$$

$$\bar{N}_g^3 \bar{\xi}_{4,g} = \frac{\bar{N}_g^3}{\bar{N}_h^3} \left[S_{4,h} (\bar{N}_h \bar{\xi}_h)^3 + 6\bar{\mu}_2 S_{3,h} (\bar{N}_h \bar{\xi}_h)^2 + (4\bar{\mu}_3 + 3\bar{\mu}_2^2) \, \bar{N}_h \bar{\xi}_h + \bar{\mu}_4 \right].$$

$$K(t) = K_h[M_i(t) - 1] = \frac{\bar{N}_g}{\bar{N}_i} \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \, (1 - M)^n \, S_{n,h} \, (\bar{N}_h \bar{\xi}_h)^{n-1}$$



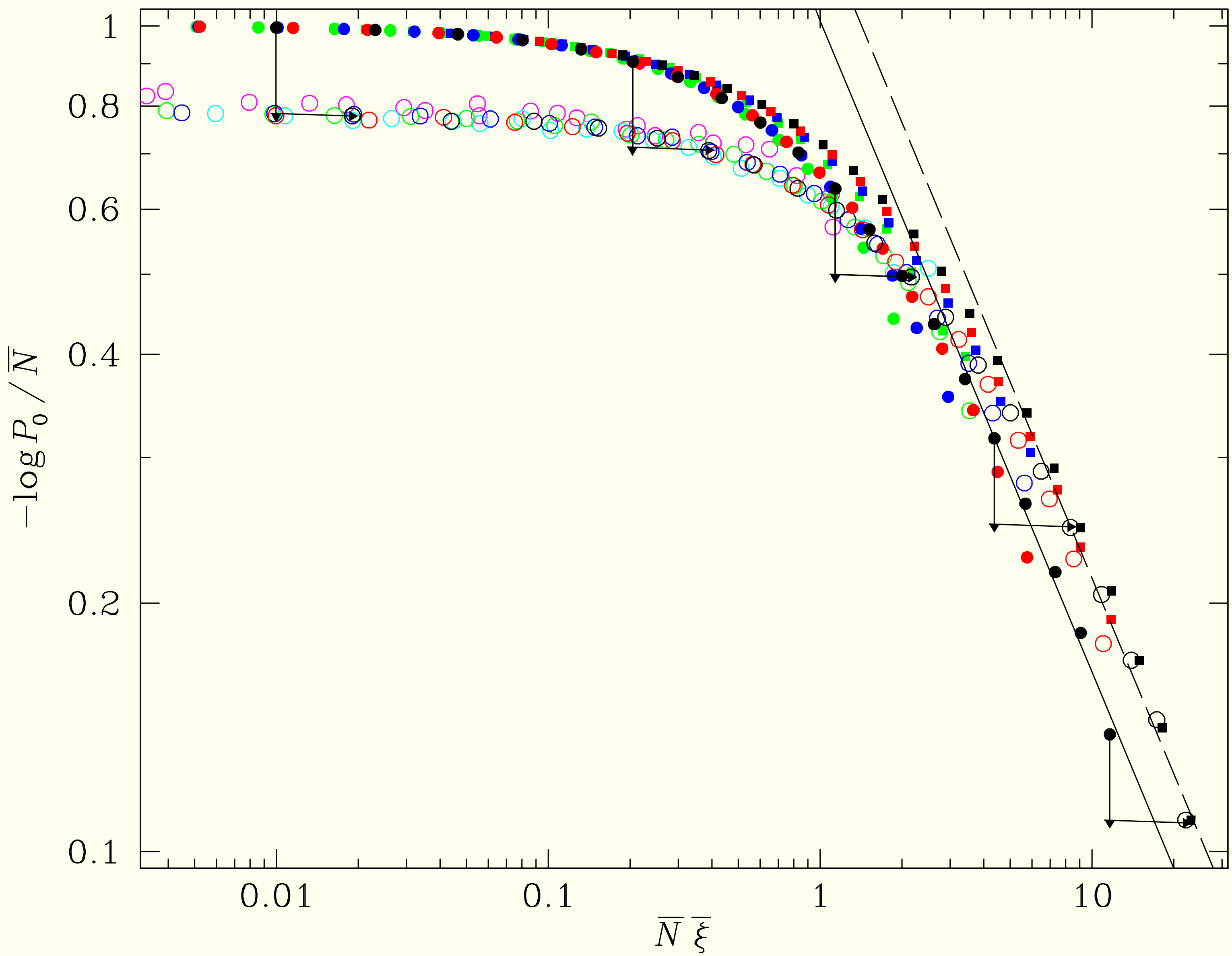
1
0.8
0.6
0.4
0.2
0
0.001
0.01
0.1
1
10
100
$-\log P_0 / \overline{N}$
$\overline{N}\,\overline{\xi}$

Point Cluster Scaling

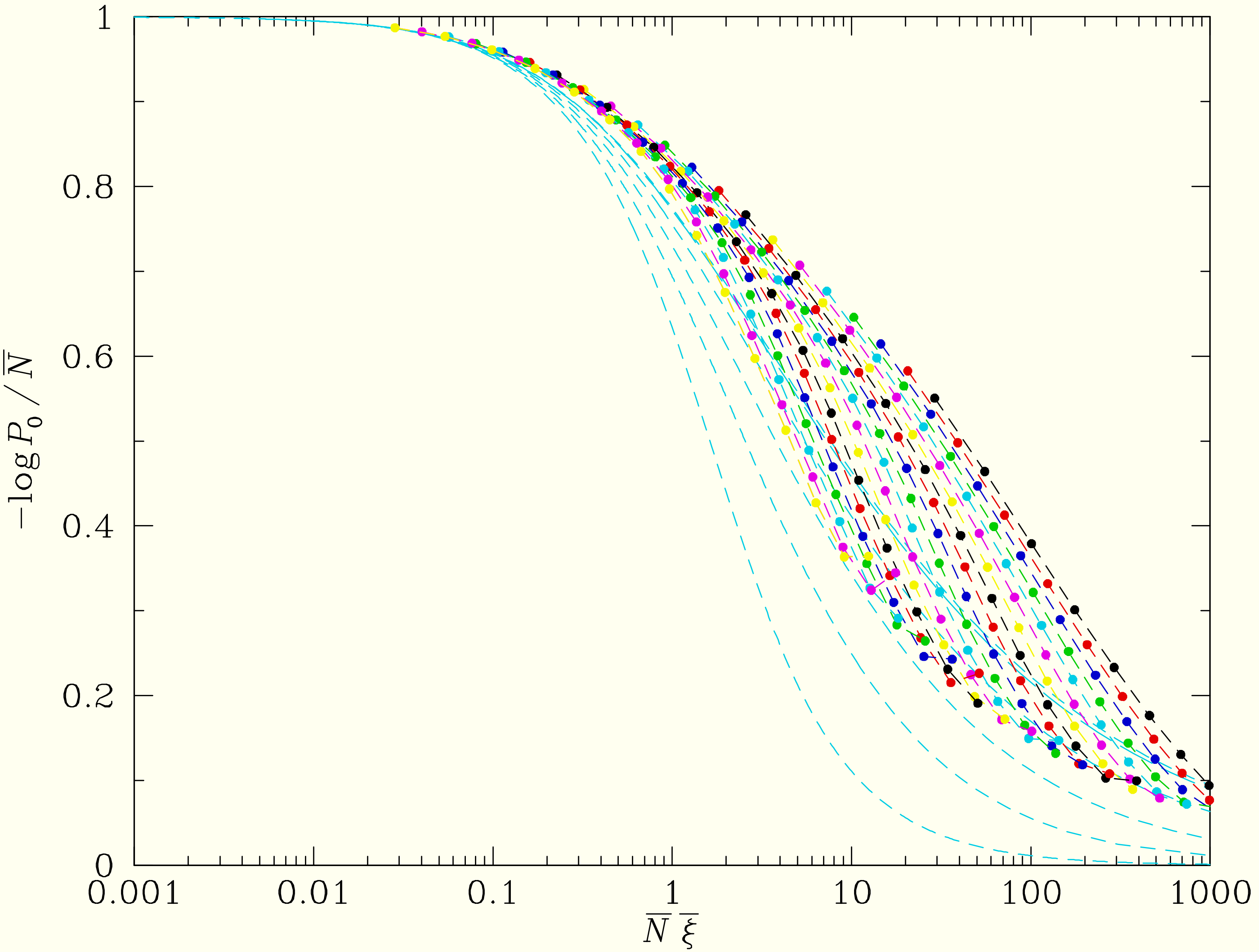
$$\frac{\chi_g}{\chi_h} = \frac{-\ln P_0/\bar{N}_g}{-\ln P_0/\bar{N}_h} = \frac{\bar{N}_h}{\bar{N}_g} = \frac{1}{\bar{N}_i} = \frac{1}{1.28}$$

$$\frac{\bar{N}_g\bar{\xi}_g}{\bar{N}_h\bar{\xi}_h} = \bar{N}_i\left(\frac{\bar{b}_g}{\bar{b}_h}\right)^2 = (1.28)(1.22)^2 = 1.90$$

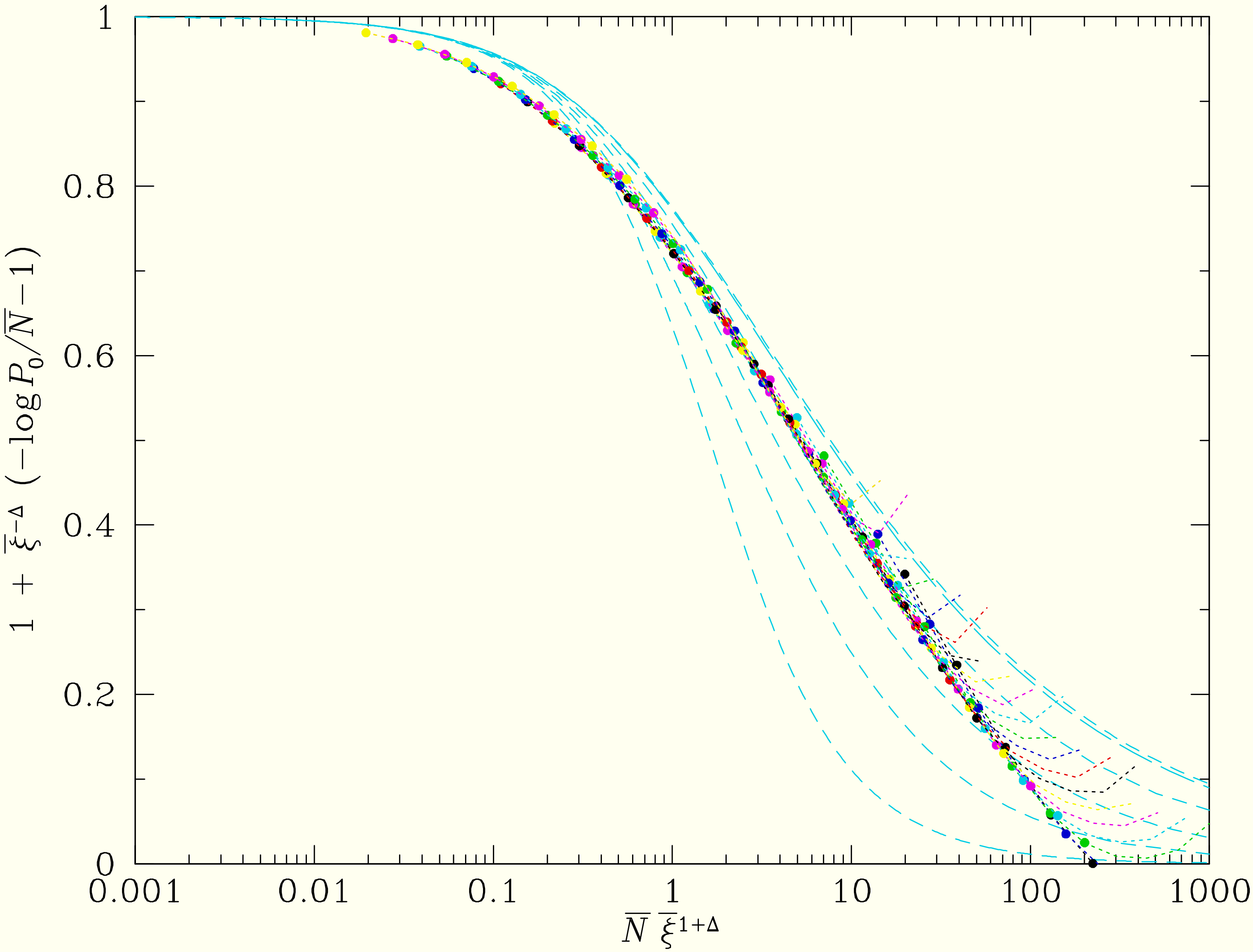
$$\chi \propto x^{-\omega} \quad \Rightarrow \quad \chi_g = \chi_h \quad @ \quad \frac{x_g}{x_h} = \frac{b_g^2}{b_h^2} \left(\frac{\bar{N}_g}{\bar{N}_h}\right)^{1-\omega}$$



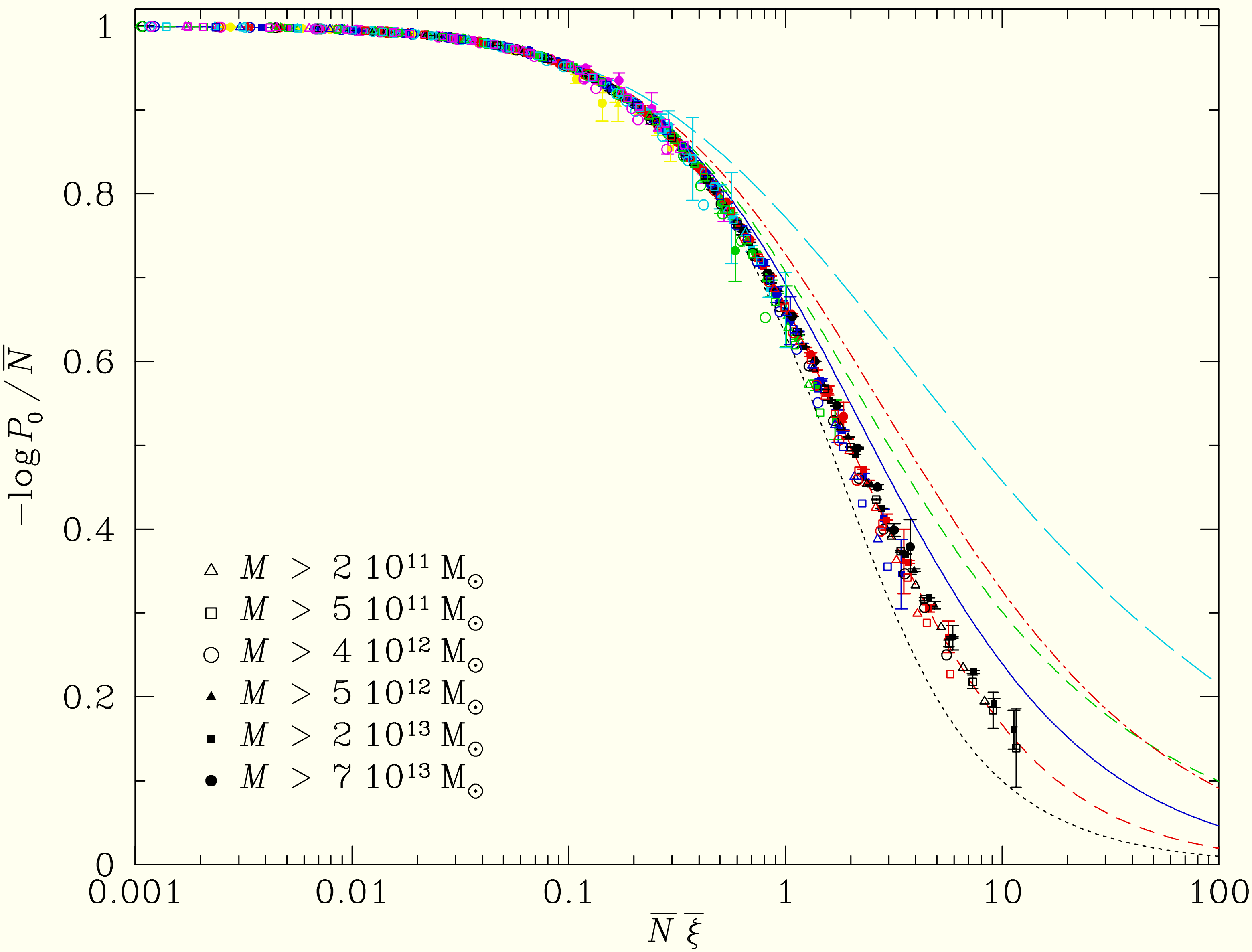
1
0.8
0.6
0.4
0.2
0.1
0.01
0.1
1
10
$-\log P_0 / \overline{N}$
$\overline{N}\,\overline{\xi}$



1
0.8
0.6
0.4
0.2
0
0.001
0.01
0.1
1
10
100
1000
$-\log P_0 / \overline{N}$
$\overline{N}\,\overline{\xi}$



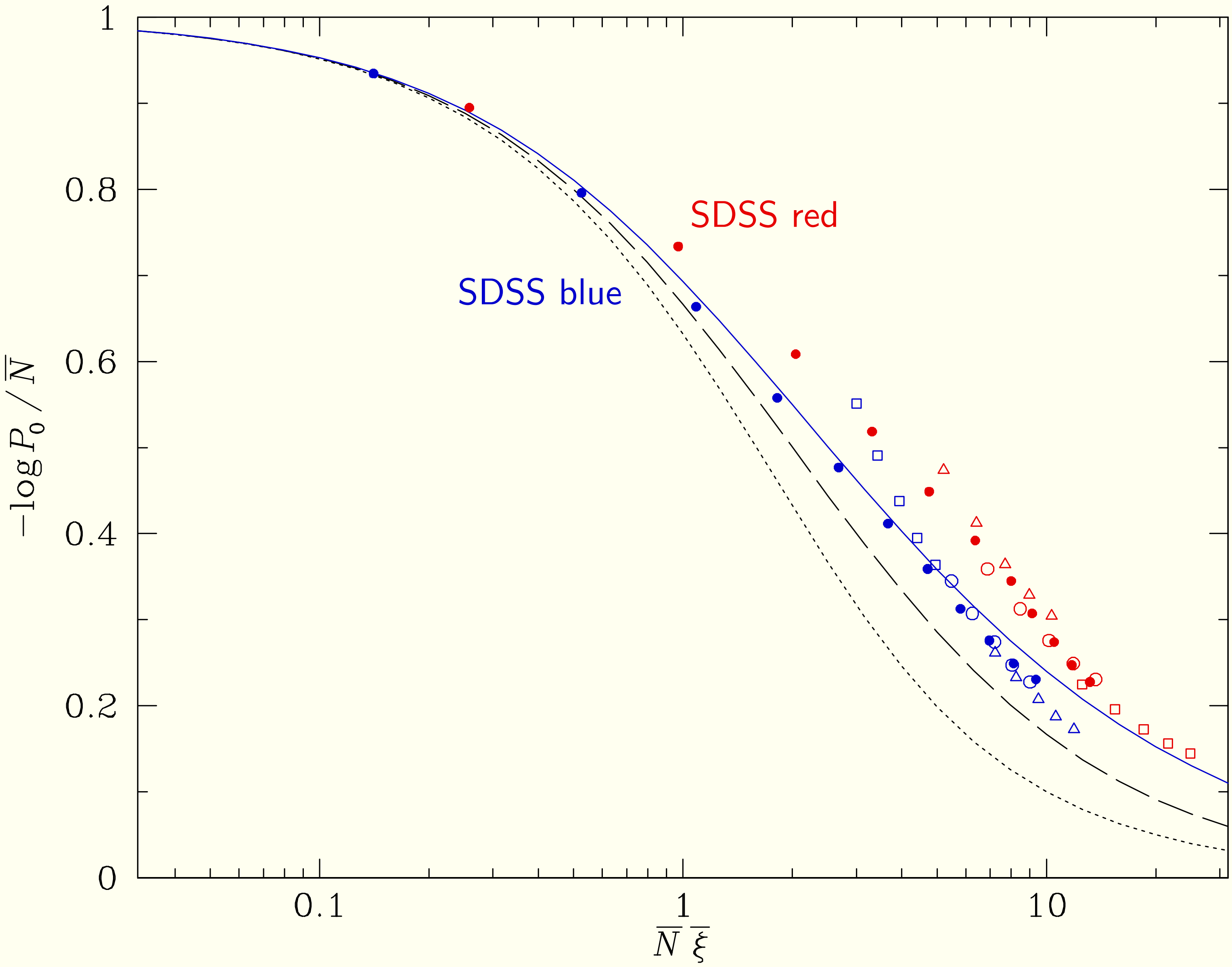
1
0.8
0.6
0.4
0.2
0
0.001
0.01
0.1
1
10
100
1000
$1 + \bar{\xi}^{-\Delta}\ (-\log P_0/\bar{N} - 1)$
$\bar{N}\ \bar{\xi}^{1+\Delta}$



1
0.8
0.6
0.4
0.2
0
0.001
0.01
0.1
1
10
100
$-\log P_0 \, / \, \overline{N}$
$\overline{N} \; \overline{\xi}$
$M > 2\ 10^{11}\ M_\odot$
$M > 5\ 10^{11}\ M_\odot$
$M > 4\ 10^{12}\ M_\odot$
$M > 5\ 10^{12}\ M_\odot$
$M > 2\ 10^{13}\ M_\odot$
$M > 7\ 10^{13}\ M_\odot$

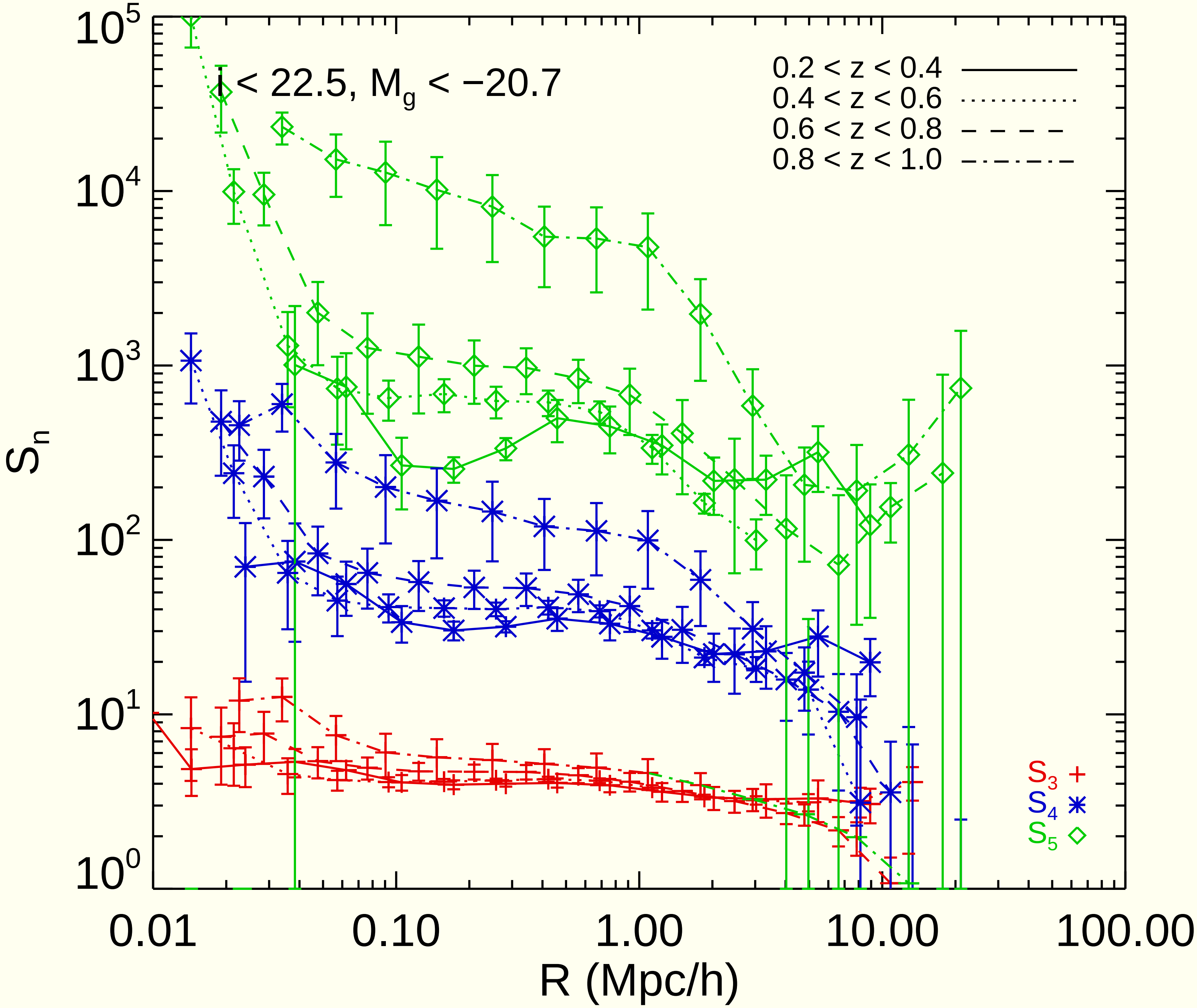
Void Statistics in Large Galaxy Redshift Surveys . . .

Tinker et al. 2008, ApJ, 686, 53



Wolk, McCracken, Colombi et al. (2013)

CFHTLS



Conclusions

- Halo model is as much about combinatorics as about shapes of things (may also be useful for hadron multiplicities)
- Halo works better than might have been expected for many nonlinear applications P_N, $S_k(\bar{\xi})$, $S_k(n)$ (at least for numerical simulations)
- Misguided effort to attempt to achieve 1% accuracy in $P(k)$ if S_n's are wrong
- Differences between mass and number statistics on small scales are substantial, not well expressed as "bias"
- Void scaling relates different samples; shift depends mostly on bias, less so on number density
- Data as a function of redshift. mass function dn/dm depends exponentially on σ, $\delta_c(z)$, $\Delta_{\rm vir}(z)$. evolution may tell us about Dark Energy

Figure 2. Non-conditional simulation of a diamond deposit