Curvature Radiation from a quantum-electrodynamics point of view

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July 5, 2016

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Part I

High energy radiation mechanisms in pulsar magnetospheres
A pair production factory

Radiation processes, a QED zoo:

- Synchrotron
- Curvature radiation
- Quantum transitions (LMXBs, Maser?...)
- Compton and Inverse Compton scatterings
  - $\gamma + \gamma \rightarrow e_+ + e_-$
  - $\gamma + \vec{B} \rightarrow e_+ + e_-$
  - ...

\[ \gamma + \gamma \rightarrow e_+ + e_- \]
Journée du LUTH - Curvature radiation in QED

High energy radiation mechanisms in pulsar magnetospheres

Historical review

Synchrotron
Schwinger 1949

States of the electron in a straight field
Huff 1931...

Spin 0 Synchrotron
Schwinger 1954, 1978...

Spin 1/2 Synchrotron
Sokolov & Ternov 1968...

Computation of low level transitions
Harding & Preece 1987...

Synchro-curvature
Cheng & Zhang 1996,
Harko & Cheng 2002,
Viganò et al. 2014
Kelner et al. 2015...

Curvature radiation
Ruderman & Sutherland 1975...

Heuristic quantum corrections
to synchro-curvature
Zhang & Yuan 1998...

States of the electron in a circular magnetic field
Voisin et al, Paper I, 2016 in prep

Spin 1/2 Curvature Radiation
Voisin et al, Paper II, 2016 in prep

History

No curvature effect

Medium curvature effect

Dominant curvature effect

Classical theory

Quantum theory
Curvature radiation: classical picture

Proposed by Ruderman and Sutherland (1975).

- Classical curvature peak: \( E_{\text{max}} = \gamma^3 \hbar c / \rho \sim 20\gamma^3 \) GeV
- Total power radiated: \( W = \frac{e^2 c}{6\pi \epsilon_0 \rho^2} \gamma^4 \sim 2 \cdot 10^8 \gamma^4 \) GeV/s
- Emission time scale \( \tau_e \sim E_{\text{max}} / W = 10^{-7} \) s
- But unphysical trajectory: \( \vec{v} \parallel \vec{B} \Rightarrow \vec{v} \wedge \vec{B} = 0 \)

Parameters: \( \gamma \sim 10^7, B \sim 10^8 \) T, \( \rho \sim 10^4 \) m
Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002), Kelner et al. (2015), Viganò et al. (2014)). The last gives:

The maximum Lorentz factor:

\[ \gamma_{\text{max}} = \left( \frac{3}{2} \frac{E_{\|} \rho}{e} \right)^{1/4} \]  \hspace{1cm} (1)

The pitch angle:

\[ \sin \alpha = \sin \alpha_0 \exp \left( -\frac{t}{\tau_\alpha} \right) \]  \hspace{1cm} (2)

With:

\[ \tau_\alpha = \frac{\gamma_{\text{max}} m_e c}{e E_{\|}} \]  \hspace{1cm} (3)

Figure: A particle pushed by an electric field \( E_{\|} \), along a magnetic field line of curvature radius \( \rho \).
Synchro-curvature radiation

This regime has been studied by several authors (Cheng and Zhang (1996), Harko and Cheng (2002), Kelner et al. (2015), Viganò et al. (2014)). The last gives:

The maximum Lorentz factor:

$$\gamma_{\text{max}} \simeq 2 \cdot 10^7 E_{\parallel 12}^{1/4} \rho_4^{1/4}$$ (4)

The pitch angle:

$$\sin \alpha = \sin \alpha_0 \exp \left( -\frac{t}{\tau_\alpha} \right)$$ (5)

With:

$$\tau_\alpha = 2 \cdot 10^{-8} \gamma_{\text{max}} E_{\parallel 12}^{-1} \text{ s}$$ (6)

Figure: A particle pushed by an electric field $E_{\parallel}$, along a magnetic field line of curvature radius $\rho$. 
But... Energy levels become quantized

Energy states of an electron in a straight magnetic field:

$$E = \sqrt{m^2c^4 + \hbar \omega_c mc^2 n} + (cp_{||})^2$$

Perpendicular momentum Parallel momentum

"Pitch angle" corresponding to the first Landau level:

$$\sin \alpha_1 \sim 2 \cdot 10^{-9} B_8 \gamma_7^{-1}$$

The first Landau state is reached in 10 meters!
Part II

State of an electron in a circular magnetic field (Voisin et al., Paper I, 2016, in prep.)
Symmetries of an electron in a circular magnetic field

- Circular static magnetic field.
- Locally homogeneous on a length scale:
  \[ \lambda = \left( \frac{2\hbar}{eB} \right)^{1/2} \approx 10^{-12} B^{-1/2}_8 \text{ m} \]
- Invariance by rotation around the axis of the circle and the magnetic field.
Equation and Approximation

Find proper states of the following set of operators:

\[ \hat{J}_x \text{ and } \hat{J}_\theta \]

\[ \hat{H}/c = \alpha^r(-i\hbar \partial_r) + \alpha^\theta(-i\hbar \partial_\theta) + \alpha^\phi(-i\hbar \partial_\phi + eA_\phi(r)) + \beta mc \]

We seek solutions at zeroth order in \( \epsilon \):

\[ \epsilon = \frac{\lambda}{\rho} \simeq 10^{-16} B_8^{-1/2} \rho_4^{-1} \]
States of an electron in a circular magnetic field:

\[
\Psi_{n,l_{\perp},l_{\parallel},a}(r, \theta, \phi) \propto e^{-\frac{r^2}{2\lambda^2}} e^{i\theta l_{\parallel}} e^{i\phi l_{\perp}} \left[ e^{i\phi l_{\perp}} L_{n-l_{\perp}}^l \left( \frac{r^2}{\lambda^2} \right) \chi^a_{\uparrow}(\theta) + L_{n-l_{\perp}}^{l_{\perp}+1} \left( \frac{r^2}{\lambda^2} \right) \chi^a_{\downarrow}(\theta) \right]
\]

(11)

\[
E = \sqrt{m^2c^4 + \hbar\omega_c mc^2n + (\hbar\Omega)^2 l_{\parallel}^2}
\]

(12)

With the pulsation around the circle: \( \Omega = c/\rho \)

- \( n \): main perpendicular quantum number.
- \( l_{\perp} + 1/2 \): angular momentum around the magnetic field (\( \hat{J}_\theta \)).
- \( l_{\parallel} \): angular momentum around the axis of the circle (\( \hat{J}_x \)).
- \( a \): spin orientation
Part III

Radiation of an ultra-relativistic electron in a circular magnetic field (Voisin et al., Paper II, 2016, in prep.)
Defining curvature radiation from a quantum point of view

Proposition of definition

Curvature radiation results of transitions between the most localized quantum states that allow all spin orientations.

States concerned

▷ Fundamental state:  $l_\parallel, n = 0, l_\perp = 0, a = 1$, spin anti-aligned.

$$\psi_0 = \psi^\uparrow_0$$  \hspace{1cm} (13)

▷ First excited state:  $l_\parallel, n = 1, l_\perp = 0, a = \pm 1$, mixed spins.

$$\cos(\xi)\psi_{1,a=1} + \sin(\xi)\psi_{1,a=-1} = \underbrace{\psi^\uparrow_1}_{l_\perp+1}(\xi) + \underbrace{\psi^\downarrow_1}_{l_\perp}(\xi)$$  \hspace{1cm} (14)
Allowed transitions and main rates

Orthogonal quantum number $n$

<table>
<thead>
<tr>
<th>Orthogonal quantum number $n$</th>
<th>Rate $r \propto \frac{B}{B_c} \gamma^4$</th>
<th>New!</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_0$</td>
<td>$\psi_1$ $a = +1$</td>
<td></td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>$\psi_1$ $a = -1$</td>
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<td></td>
</tr>
</tbody>
</table>

Energy (mostly $l_{||}$)

Rate $r \propto \gamma^4$ Classical

Rate $r \propto \frac{B}{B_c} \gamma^4$
Spin loves flip-flops if $B \geq B_c = 4.4 \cdot 10^9 \text{T}$

The transition rates read:

\[
\frac{d\omega_{fi}}{d\omega} = \left\|M_{fi}\right\|^2 \frac{2\pi \hbar \delta (E_f + \hbar \omega - E_i)}{c^3(2\pi)^3 / V} \frac{\omega^2 d\omega}{(2\pi)^3 / V}
\]

\[
\left\|M_{fi}\right\|^2 \propto \left\| \int \overline{\Psi_f} \gamma^\mu e_\mu \Psi_i e^{-i\vec{k} \cdot \vec{x}} d^3x \right\|^2
\]

Energy (mostly $l_\parallel$)

\[
\propto \frac{B}{B_c} \gamma^4
\]

Rate depends on the initial spin mixture
Spectrum expressions

Classical curvature radiation is identical to its classical version up to high energy corrections. Spin-flip expressions are very similar.

\[
\begin{align*}
\frac{d^2I_{\parallel}}{d\omega \, d\Omega} &= \frac{1}{2\pi\Omega} \frac{e^2\omega^2}{12\pi^3\epsilon_0 c} \left( \kappa^2 + \frac{1}{\tilde{\gamma}^2} \right)^2 K_{2/3}(\xi) \\
\frac{d^2I_{\perp}}{d\omega \, d\Omega} &= \frac{1}{2\pi\Omega} \frac{e^2\omega^2}{12\pi^3\epsilon_0 c} \kappa^2 \left( \kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) K_{1/3}(\xi) \\
\frac{d^2I_{\parallel}}{d\omega \, d\Omega} &= \frac{1}{2\pi\Omega} \frac{e^2\omega^2}{12\pi^3\epsilon_0 c} \left( \kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) \frac{1}{2\tilde{\gamma}^2} \frac{B}{B_c} K_{2/3}(\xi) \\
\frac{d^2I_{\perp}}{d\omega \, d\Omega} &= \frac{1}{2\pi\Omega} \frac{e^2\omega^2}{12\pi^3\epsilon_0 c} \left( \kappa^2 + \frac{1}{\tilde{\gamma}^2} \right) \frac{1}{2\tilde{\gamma}^2} \frac{B}{B_c} K_{1/3}(\xi)
\end{align*}
\]

Polarizations and symbols above are within the conventions of Jackson (1998)
Radiation of an ultra-relativistic electron in a circular magnetic field
Radiated intensities

Spin-flip curvature radiation is mostly unpolarized:

\[
\begin{align*}
I_{\parallel \downarrow \downarrow} &= \frac{1}{\epsilon_0 c} \left( \frac{\Omega e}{16\pi} \right)^2 \frac{28}{3} \tilde{\gamma}^4 \quad (17) \\
I_{\perp \downarrow \downarrow} &= \frac{1}{\epsilon_0 c} \left( \frac{\Omega e}{16\pi} \right)^2 \frac{4}{3} \tilde{\gamma}^4 \quad (18) \\
I_{\parallel \uparrow \downarrow} &= \frac{1}{\epsilon_0 c} \left( \frac{\Omega e}{16\pi} \right)^2 \frac{16}{3} \frac{B}{B_c} \tilde{\gamma}^4 \quad (19) \\
I_{\perp \uparrow \downarrow} &= \frac{1}{\epsilon_0 c} \left( \frac{\Omega e}{16\pi} \right)^2 \frac{16}{3} \frac{B}{B_c} \tilde{\gamma}^4 \quad (20)
\end{align*}
\]

Polarizations and symbols above are within the conventions of Jackson (1998)
Corrections at high energies

Figure: Effect of quantum corrections at high energies obtained by the replacement $\hbar \omega \rightarrow \hbar \omega \left(1 + \frac{\hbar \omega}{E}\right)$. Here Lorentz factor $= 2 \cdot 10^7$. 
Conclusion

- We prove that curvature radiation can be derived in a fully consistent way using quantum electrodynamics in a high-magnetic-field, ultra-relativistic regime.
- We show that spin-flip transitions amount to 10% in "normal" $10^8$ Tesla pulsars and dominate for magnetic fields $\gtrsim B_c = 4.4 \cdot 10^9$ Teslas. Spin-flip curvature radiation should not be neglected in young pulsars (Crab-like), and becomes dominant in magnetars.
- For both we worked out high energy quantum corrections.
- The spectra are very similar in shape to the usual curvature/synchrotron but differ in intensity (for spin-flip).
- Contrary to constant-spin (classical) radiation, spin-flip curvature radiation is mostly unpolarized.
- In the longer term, "quantum synchro-curvature radiation" should also be investigated.


S. R. Kelner, A. Yu Prosekin, and F. A. Aharonian. Synchro-curvature radiation of charged particles in the strong...
